WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-I Examinations, 2018

Physics-Honours

Paper-PHSA-I

Time Allotted: 4 Hours — Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Use separate answer scripts for each Unit.

Unit-IA

Question No. 1 is compulsory and answer other questions from Group A and Group B according to the instructions

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) 'The logarithm of a complex number $z = re^{i\theta}$ is a multivalued function' —Explain.

(b) Is the given series $1 - \frac{1}{2} + \frac{1}{8} \cdots\cdots$, convergent or divergent?

(c) If $f(x) = Ae^{\alpha x}$ and $g(x) = Be^{\beta x}$ where $\alpha \neq \beta$, then show that $f(x)$ and $g(x)$ are linearly independent.

(d) Find a vector $\vec{\delta}$ which is perpendicular to both $\vec{\alpha} = 4\hat{i} + 5\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{i} - 4\hat{j} + 5\hat{k}$ and satisfy $\vec{\delta} \cdot \vec{\gamma} = 2$, where $\vec{\gamma} = 3\hat{i} + \hat{j} - \hat{k}$.

(e) Check the singularity of the following equation at $x = 0$ and state the type of singularity: $x^3 \frac{d^2y}{dx^2} - 6y = 0$.

(f) For a system of particles, if the total linear momentum is conserved, then show that the centre of mass is either at rest or moving uniformly.

(g) Show that the trace of a matrix remain invariant under similarity transformation.

(h) How is Coriolis force different from centrifugal force?

Group-A

Answer any *three* questions from the following 10×3 = 30

2. (a) Write down the expression of Binomial distribution function for a random variable *X*. Calculate its mean and variance from this distribution. 1+2+2

(b) Express $\frac{\partial^2}{\partial x^2}$ in spherical polar co-ordinate system. 3

(c) Show that for any vector $\vec{A} \cdot \frac{d\vec{A}}{dt} = A\frac{dA}{dt}$. 2

3. (a) Show that $\vec{\nabla}(\phi \vec{A}) = (\vec{\nabla}\phi) \cdot \vec{A} + \phi \vec{\nabla} \cdot \vec{A}$. Hence prove that $\vec{\nabla} \cdot \left(\frac{\vec{r}}{r^3}\right) = 0$. 3+2

(b) Use Green's theorem in a plane to show that the area bounded by a simple closed curve is $\frac{1}{2}\oint_C (x\,dy - y\,dx)$. 2

(c) Define Dirac delta-function and show that a function $f(x)$ may be expressed as $f(x) = \int_{-\infty}^{\infty} f(x')\,\delta(x - x')dx'$. 1+2

4. (a) Consider the differential equation: $\frac{d^2x}{dt^2} + \omega^2 x = 0$. Assuming a power series solution $x(t) = \sum_n a_n t^{n+k}$, establish the indicial equation: $k(k-1) = 0$. Hence show that the two series solutions converge to the forms $\cos\omega t$ and $\sin\omega t$. 3+4

(b) Solve by the method of separation of variables 3

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(x,\ 0) = 4e^{-x}.$$

5. (a) Expand $f(x) = 1,\ \ 0 < x < \pi$ in a Fourier series and hence show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$. 5+2

(b) Find the Fourier transform of $e^{-x^2/2}$. 3

6. (a) The Legendre polynomial $P_n(x)$ is obtained from the expression $(1 - 2xt + t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x)t^n$. Using this, establish that $nP_n(x) - (2n-1)xP_{n-1}(x) + (n-1)P_{n-2}(x) = 0$. 3

(b) If S and A are unitary matrices, then show that $S^{-1}AS$ is also unitary. 2

(c) Show that all the diagonal elements of a Hermitian matrix are necessarily real. 2

(d) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$. 3

Group-B

Answer any *one* question from the following 10×1 = 10

7. (a) Show that the radial and transverse components of acceleration in polar coordinates are given by $a_r = \ddot{r} - r\ddot{\theta}^2,\ a_\theta = r\ddot{\theta} + 2r\dot{\theta}$. 3

(b) The relative velocity of two particles having masses m_1 and m_2 is $\vec{v}$ and the velocity of their centre of mass is $\vec{u}$. If M is the total mass and μ is the reduced mass, show that the total kinetic energy is $T = \frac{1}{2}Mu^2 + \frac{1}{2}\mu v^2$. 3

(c) Prove that total mechanical energy of a particle in a conservative force field remains constant if the potential energy is not an explicit function of time. 3

(d) Why do we need the concept of radius of gyration of a given body? 1

8. (a) Write down Galilean transformation and show that Newton's 2nd law of motion is invariant under this transformation. 1+2

(b) What do you mean by 'Coriolis force'? Calculate its action on a vertically falling body. 1+3

(c) For a homogeneous cube of mass 'M' and side 'a', estimate the moments of inertia I_{xx}, I_{yy}, I_{zz} where the origin at the centre of mass. 3

Unit -IB

Question No. 9 is compulsory and answer other questions from Group C and Group D according to the instructions.

9. Answer any ***five*** questions from the following: 2×5 = 10

(a) Write down with suitable diagram the reciprocity theorem of bending of beams.

(b) Calculate the amount of work done if a soap bubble is slowly enlarged from a radius of 10 cm to a radius of 20 cm. Given, S = 30 dynes/cm.

(c) What are the basic assumptions behind the validity of Bernoulli's theorem of fluid motion?

(d) Show that the velocity of longitudinal waves is greater than the velocity of transverse waves in solids.

(e) Show that the motion of a particle under centro symmetric central force is planar.

(f) Prove that Kepler's first and second laws lead to the conservation of angular momentum.

(g) What is shaspness of resonance? Explain graphically.

(h) Show that for a non dispersive medium the phase velocity and group velocity of a progressive wave are equal.

Group-C

Answer any *two* questions from the following 10×2 = 20

10.(a) Two bodies of masses m_1 and m_2 are at a distance d from each other. Show that the potential at a point where the gravitational intensity is zero is given by $V = -\frac{G}{d}(m_1 + m_2 + 2\sqrt{m_1 m_2})$. 4

(b) Considering the central force of interaction between two particles of masses m_1 and m_2, establish the equation of motion of the system and find out an expression for reduced mass. 3

(c) A vector is defined as $\vec{A} = k(\vec{L} \times \vec{p}) = \hat{r}$. Where L and p denote angular and linear momentum respectively. Estimate $\vec{A} \cdot \vec{r}$. Hence establish the equation of Keplar's elliptical orbit *i.e.* $\frac{1}{r} = \frac{1}{kL^2}(1 - a\cos\theta)$. Here, $a = |\vec{A}|$ and θ is the angle between $\vec{A}$ and $\vec{r}$. 2+1

11.(a) Establish the expression for bending moment of a rectangular beam. 5

(b) Write down the equation of continuity in case of fluid motion and explain its significance. 2

(c) A capillary tube of radius a and length l is fitted horizontally at the bottom of a cylindrical flask of cross-section A. Initially, there is water in the flask up to a height h_1. Show that the time $T = \frac{8\eta l A}{\pi \rho g a^4} \ln \frac{h_1}{h_2}$ is required for the height to reduce from h_1 to h_2. Here $\eta =$ viscocity of water. 3

12.(a) Establish the relation between surface tension and surface energy. 5

(b) Obtain Bernoulli's theorem for ideal fluid from conservation of energy principle. 3

(c) Water flows along a horizontal tube of which the cross section is not constant. Apply Bernoulli's theorem to calculate the change in pressure when the velocity of water flow changes from 10 cm/sec to 20 cm/sec. 2

Group-D

Answer any *two* questions from the following 10×2 = 20

13.(a) For an under-damped oscillation, set up and solve the differential equation. Hence show that the logarithm of the ratio of successive amplitudes is constant. 1+3+2

(b) If ω_1 and ω_2 are half-power frequencies in forced vibration and n is the frequency when no damping is there, show that $\omega_1\omega_2 = n^2$. 4

14.(a) Show that the velocity of acoustic waves travelling along a solid rod is given by $\sqrt{\frac{Y}{d}}$, where Y is the Young's modulus and d is the density. Mention the assumption made. 4

(b) Show that the velocity C of sound in a gas is given by $C = \left(\frac{\gamma}{3}\right)^{1/2} v_r$, where v_r is the r.m.s. speed of the gas molecules and r is the ratio of the two specific heats. 2

(c) The phase velocity v of deep water waves of wave length λ is given by $v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi S}{\rho\lambda}$, where g = acceleration due to gravity, S = surface tension and $\rho =$ density of water. Determine the wave length λ_0 of the waves which do not disperse in water. Show that, for $\lambda << \lambda_0$, the group velocity v_g is $\frac{3}{2}v$. 2+2

15.(a) For a vibrating stretched string rigidly fixed at both ends, the displacement y at any point x at any instant of time t is given by: 4

$$y(x,\, t) = \sum_{n=1}^{\infty} C_n \sin K_n x \cos(W_n t - \phi_n) \quad \text{where, } K_n = \frac{n\pi}{l} \text{ and } W_n = \frac{n\pi c}{l}.$$

Show that the total energy of the string is $E = \frac{M}{4}\sum_{n=1}^{\infty} W_n^2 C_n^2$, where M is the mass of the string.

(b) Explain why the sound produced by a struck string is more melodious than that produced by a plucked string. 2

(c) Establish the expression for modified frequency when a sound is moving at uniform speed towards a stationary observer. 2

(d) Explain what is meant by 'bel' and 'phone'. 2

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-I Examinations, 2018

PHYSICS-HONOURS

PAPER-PHSA-II-A

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *four* questions from the rest, taking at least one question from Group-A

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Let $f(u)\,du$ be the probability that a gas molecule has *x*-component of velocity between *u* and *u* + *du*. Plot the nature of variation of $f(u)$ with *u* at two different temperatures T_1 and T_2.

(b) Show that the law of equipartition of energy demands that the motion of relatively large Brownian particles are practically unnoticed.

(c) Calculate the pressure exerted by one mole of a van-der Waal's gas at a temperature *T* = 273K, the volume of the gas being 0.55 litre.
(a = 0.37 N-m^4-mol^{-2}, b = 43×10^{-6} m^3 mol^{-1}, R = 8.31 J $mol^{-1}K^{-1}$)

(d) Calculate the work done by 1 mole of gas during a quasistatic isothermal expansion from a volume V_i to a volume V_f, whose equation of state is given by $pV = RT\left(1-\frac{B}{V}\right)$, where $B = f(T)$.

(e) Show that the relation $\eta = \frac{1}{3}mn\bar{c}\lambda$ implies that η is independent of *n* and hence pressure at a given temperature (The symbols have than usual meanings).

(f) Distinguish between the free expansion and Joule-Thomson expansion.

(g) How much work must be done to extract 10 J of heat from a reservoir at 7°C and transfer it to one at 27°C by means of a refrigerator working in a Carnot Cycle?

(h) Starting from the expression of Helmholtz free energy $F(T,V)$, show $C_V = -T\left(\frac{\partial^2 F}{\partial T^2}\right)_V$.

Group-A

2. (a) Maxwell's energy distribution law is given by the relation $n(\varepsilon)d\varepsilon = A\sqrt{\varepsilon}\, e^{-\varepsilon/kT}$, where $n(\varepsilon)d\varepsilon$ is the number of gas molecules having energy between ε and $\varepsilon + d\varepsilon$. Calculate the normalization constant *A*, where the total number of molecules is *N*. Also find the average and most probable energy of the system. ($k \to$ Boltzmann constant). 3+2+2

(b) Show that if the most probable speed is taken as the unit of speed for gas molecules, then the speed distribution becomes independent of temperature. 3

3. (a) Define the critical constants of a gas. Derive the expressions of three critical constants for a real gas following the relation $P(V-b)=RTe^{-a/RTV}$ in terms of '*a*' and '*b*' where, '*a*' and '*b*' are two constants. Show that in this case the value of critical coefficient is $\frac{e^2}{2}$. 3+3+1

   (b) Find the expression of reduced equation of state for the above case. 2

   (c) What is law of corresponding states? 1

Group-B

4. (a) The equation of state of an ideal elastic substance of length *L* is $F=CT\left(\frac{L}{L_0}-\frac{L_0^2}{L^2}\right)$, (*F*, the tension) where *C* is constant and L_0 (the value of *L* at zero tension *F*) is the function of temperature only. Show that the isothermal Young modulus $Y=\frac{F}{A}+3CT\frac{L_0^2}{AL^2}$, where *A* is the cross section of the substance. Find the work required to change *L* from L_0 to $\frac{1}{3}L_0$ quasistatically and isothermally. 2+3

   (b) If E_S and E_T are the adiabatic and isothermal elasticity constants then show that $\frac{E_S}{E_T}=r$. 3

   (c) The efficiency of a Carnot engine can be increased by increasing the temperature of source or by decreasing the temperature of sink. Discuss which one is better. 2

5. (a) Show that the entropy of *n* moles of an ideal gas of molar specific heat at constant volume (C_V) at temperature *T* and volume *V* is given by $S=nC_V\ln T+nR\ln V+S_0$, assuming C_V is independent of temperature. 3+3

   (b) Show that entropy increases when two gases at the same temperature and pressure diffuse into each other. Discuss Gibb's paradox in this connection. 2+2

6. (a) What do you mean by inversion temperature of a gas? The equation of state of real gas is $P(V-b)=RT\,\exp(-a/RTV)$, where *a* and *b* are constants. Find the inversion temperature of gas. 1+3

   (b) The Gibbs' free energy of a system is given by $G=RT\ln[ap/(RT)^{3/2}]$, where *a* is a constant, *T*, the temperature and *p*, the pressure and *R*, the universal gas constant. Find the entropy and specific heat at constant pressure of the system. 4

   (c) Draw the solid-liquid-gas phase diagram of water and indicate the triple point. 2

7. (a) Define thermal conductivity and write down its S.I. unit. 1+1

   (b) Find an expression for the rate of radial flow of heat through a hollow cylinder whose inner and outer surfaces are at distances r_1 and r_2 respectively and maintained at different temperatures. What will be the temperature at a radial distance $r\,(r_1<r<r_2)$? 3

   (c) Establish Newton's law of cooling from Stefan's law. 2

   (d) What is the solar constant? Use Stefan's law to estimate the rate at which the solar energy is reaching the top of Earth's atmosphere. Assume the sun to be a black body at temperature 5800 K. 1+2

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