WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-II Examinations, 2018

MATHEMATICS-HONOURS

PAPER-MTMA-III

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Group-A

Answer any ***three*** questions from the following 5×3 = 15

1. Define special roots of $x^n - 1 = 0$. Prove that the special roots of the equation $x^n - 1 = 0$ are the roots of a reciprocal equation. 2+3

2. Solve by using Cardan's method: 5

   $x^3 + 3x^2 - 3 = 0$.

3. Solve by Ferraris method: 5

   $x^4 + 3x^3 + x^2 - 2 = 0$.

4. (a) If x, y, z are positive rational numbers then show that 3

   $$\left(\frac{x^2+y^2+z^2}{x+y+z}\right)^{x+y+z} \geq x^x y^y z^z .$$

   (b) If a, b, c are three positive numbers in harmonic progression and n is a positive integer greater than 1, prove that 2

   $$a^n + c^n > 2b^n .$$

5. Reduce the reciprocal equation 5

   $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$ to the standard form and solve it.

6. (a) Find greatest value of $(2x+1)^3(y+2)^2$ when $x+y=3$ and $-\frac{1}{2}<x<5$. 3

(b) If a, b, c are unequal positive numbers such that sum of any two numbers is greater than the third then show that 2

$$\frac{1}{b+c-a}+\frac{1}{c+a-b}+\frac{1}{a+b-c}>\frac{9}{a+b+c}.$$

Group-B

Answer any ***one*** question from the following 10×1 = 10

7. (a) Show that if the permutation $\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 3 & 4 & 2\end{pmatrix}$ multiplied three times itself, then it will give an identity permutation. 2

(b) If $a=(1\ 2\ 3\ 4)$ then show that the set $\{a, a^2, a^3, a^4\}$ forms a cyclic group. 3

(c) Show that every proper subgroup of symmetric group S_3 is cyclic. 3

(d) Find the images of the element 3 and 4 if $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 2 & 3 & & & 1 & 4\end{pmatrix}$ be an even permutation. 2

8. (a) If G be a group and H be a subgroup of G then prove that any two left cosets of H in G are either identical or they have no common element. 3

(b) Prove that every group of prime order is cyclic. 3

(c) Express $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 6 & 5 & 2 & 1 & 3 & 4\end{pmatrix}$ as a product of transpositions. Find also order of the permutation. 2

(d) Show that if two right cosets Ha and Hb be distinct then two left cosets $a^{-1}H$ and $b^{-1}H$ are distinct. 2

Group-C

Answer any ***two*** questions from the following 10×2 = 20

9. (a) Let P stands for the set of all the functions from [0, 1] into ℝ, which are differentiable over [0, 1]. Show that P is a vector space over ℝ if addition of functions in P and multiplication of functions in P by the elements of ℝ are defined pointwise on [0, 1]. 3

(b) Find a basis and dimension of the subspace W of $\mathbb{R}^3$ where 2+1

$$W=\{(x,\ y,\ z)\in\mathbb{R}^3:\ \ x+y+z=0\}.$$

(c) State Cayley-Hamilton theorem. Hence compute A^{-1} where $A=\begin{pmatrix}2 & 1\\ 3 & 5\end{pmatrix}$. 1+3

10.(a) Correct or Justify: Let V be a vector space over a field F. Let U and W be two subspaces of V such that dim U = dim W. Then U = W. 3

(b) Prove that in an Euclidean vector space, $\|\alpha+\beta\| \le \|\alpha\| + \|\beta\|$. 3

What happens when the equality holds?

(c) Use Gram-Schimdt process to obtain an orthonormal basis of the subspace of the Euclidean space $\mathbb{R}^4$ with standard inner product, generated by the linearly independent set $\{(1,1,0,1),(1,1,0,0),(0,1,0,1)\}$. 4

11.(a) Investigate for what values of a and b the following system of equations 4

$$x+y+z=1$$
$$x+2y-z=b$$
$$5x+7y+az=b^2$$

has (i) only one solution, (ii) no solution and (iii) an infinite number of solutions.

(b) Give an example to show that union of two vector subspaces of a vector space V(F) may not be a subspace of V(F). 2

(c) If in a vector space V(F) of dimension n, the set $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is a set of generators, prove that $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is basis of V. 4

12.(a) Prove that every non-zero orthogonal set of vectors of a Euclidean space is linearly independent. 3

(b) Prove that the solutions of a homogeneous system $\mathbf{AX}=\underline{\mathbf{0}}$ in n unknowns where $\mathbf{A}$ is an $m\times n$ matrix over a field F, form a subspace of $V_n(F)$. 3

(c) Find an orthogonal matrix $\mathbf{P}$ such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix, where 4

$$\mathbf{A}=\begin{pmatrix}3 & 1 & 1\\ 1 & 3 & 1\\ 1 & 1 & 3\end{pmatrix}.$$

Group-D

Answer any ***two*** questions from the following 10×2 = 20

13.(a) Prove that every bounded sequence of real numbers has a convergent subsequence. Is the result true if the word 'bounded' be replaced by 'bounded below'? Justify. 3+1

(b) Define upper and lower limits of a bounded sequence. Determine these limits for the sequence $\{a_n\}_n$ where $a_n = (-1)^n + \frac{1}{n+1}$. 3

(c) Prove that if the subsequence $\{x_{3n}\}_n$ of a monotonic sequence $\{x_n\}$ converges to l then every subsequence of $\{x_n\}$ converges to l. 3

14.(a) Examine the convergence of the series 3

$$\sum_n \left\{\frac{2\cdot 4\cdot 6\cdots 2n}{3\cdot 5\cdot 7\cdots(2n+1)}\right\}^2.$$

(b) If $\{b_n\}$ is a monotone bounded sequence and $\sum_n a_n$ is convergent series then prove that $\sum a_n b_n$ is convergent. 4

(c) Examine the convergence of the series $\sum \frac{(-1)^n}{(n+1)\log(n+1)}$. 3

15.(a) Let $f:[a, b]\to\mathbb{R}$ be continuous on a closed interval $[a, b]$ and $f(a)\cdot f(b)<0$. Prove that there exists at least one point c in the open interval (a, b) such that $f(c)=0$. 4

(b) Find a and b in order that $\lim_{x\to 0}\frac{a\sin(2x)-b\sin(3x)}{5x^3}=1$. 3

(c) Prove that $f(3)$ is a minimum value of 3

$f(x)=|3-x|+|2+x|+|5-x|,\ x\in\mathbb{R}$

but $f'(3)$ does not exist.

16.(a) State and prove Cauchy's Mean Value theorem and deduce from it Lagrange's Mean Value theorem. 1+3+1

(b) $f:\mathbb{R}\to\mathbb{R}$ is so defined that 3

$$f(x)=\begin{cases}\frac{1}{7} & \text{when } x \text{ is rational}\\ \frac{1}{17} & \text{when } x \text{ is irrational}\end{cases}$$

Prove that f is continuous at no point in $\mathbb{R}$.

(c) If a function $f(x)$ be defined as $f(x)=0$ when $x\neq 0$ and $f(x)=1$ when $x=0$, prove that there exists no function $g(x)$ such that $g'(x)=f(x)$. 2

Group-E

Answer any ***five*** questions from the following 5×5 = 25

17. Define limit point of a subset of $\mathbb{R}\times\mathbb{R}$. Show that the set $Q\times Q=\{(x,\ y)\,|\,x,\ y$ are both rational numbers$\}$ is neither open nor closed in $\mathbb{R}\times\mathbb{R}$. 1+4

18. Let $f:S\to\mathbb{R}$ be a function, where $S\subset\mathbb{R}^2$. If f is continuous at a point $(a,\,b)\in S$ then show that $f(x,\,b)$ is continuous at $x=a$ and $f(a,y)$ is continuous at $y=b$. Is the converse of the result true? Justify your answer. 5

19. Let $f:S\to\mathbb{R}$ be a function, where $S\subset\mathbb{R}^2$. What do you mean by differentiability of f at a point $(a,\,b)\in S$? Show that differentiability of f at $(a,\,b)$ implies the continuity of f at $(a,\,b)$ and the existence of first order partial derivatives at that point. 5

20. Let $f(x,\ y)=\begin{cases} x^2\sin\dfrac{1}{x}+y^2\cos\dfrac{1}{y}, & \text{when } x\neq 0,\ y\neq 0\\ x^2\sin\dfrac{1}{x}, & \text{when } x\neq 0\\ y^2\cos\dfrac{1}{y}, & \text{when } y\neq 0\\ 0, & \text{when } x^2+y^2=0.\end{cases}$ 3+2

Prove that both f_x and f_y exist at (0, 0) but none is continuous there. Examine the differentiability of $f(x,\ y)$ at (0, 0).

21. If u be a homogeneous function of $x,\ y,\ z$ of degree n having continuous second order partial derivatives and if $u=f(\xi,\eta,\zeta)$ where ξ,η,ζ are the partial derivatives of u w.r.t. $x,\ y,\ z$ respectively, prove that 5

$$\xi\frac{\partial u}{\partial\xi}+\eta\frac{\partial u}{\partial\eta}+\zeta\frac{\partial u}{\partial\zeta}=\frac{nu}{n-1},\ (n\neq 1)\ .$$

22. If $f(x,\ y)$ is a function of two variables x and y such that first order partial derivatives f_x and f_y are differentiable at an interior point $(a,\,b)$ of the domain of definition of the function then show that $f_{xy}(a,\,b)=f_{yx}(a,\,b)$. 5

23. Let $u,\ v$ be functions of α,β,γ having continuous first order partial derivatives and α,β,γ be functions of x and y having continuous first order partial derivatives. Prove that 5

$$\frac{\partial(u,v)}{\partial(x,y)}=\frac{\partial(u,v)}{\partial(\alpha,\beta)}\cdot\frac{\partial(\alpha,\beta)}{\partial(x,y)}+\frac{\partial(u,v)}{\partial(\beta,\gamma)}\cdot\frac{\partial(\beta,\gamma)}{\partial(x,y)}+\frac{\partial(u,v)}{\partial(\gamma,\alpha)}\cdot\frac{\partial(\gamma,\alpha)}{\partial(x,y)}.$$

24. Write the conditions so that the functional equation $f(x, y) = 0$ does define an implicit function. Show that the equation $y^2 - yx^2 - 2x^5 = 0$ determine uniquely implicit function in the neighbourhood of the point (1, –1). Also find the first order derivative of the solution. 2+2+1

25. Find the condition, by using Jacobian, that the expression $ax^2 + by^2 + cz^2 + 2hxy + 2fyz + 2gzx$ can be expressed as product of two linear factors. 5

Group-F

Answer any ***two*** questions from the following 5×2 = 10

26. Find the area between the curve $xy^2 = 4a^2(2a - x)$ and its asymptote. 5

27. Find the moment of inertia of a thin uniform lamina in the form of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the *x*-axis. 5

28. Determine the co-ordinates of the centre of gravity of a segment of the parabola $y^2 = ax$ cut off by the straight line $x = a$. 5

29. Find the volume of the solid generated by the revolution about *x*-axis, of two arcs intercepted by the parabola $y^2 = 8ax$ and the circle $x^2 + y^2 = 9a^2$. 5

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Group-A

Answer any ***two*** questions from the following — 10×2 = 20

1. (a) A variable plane is parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes at A, B, C respectively. Find the equation of the surface on which the circle ABC lies. 5

   (b) Show that the pole of any tangent to the hyperbola $xy=c^2$ with respect to the circle $x^2+y^2=a^2$ lies on concentric and similar hyperbola. 5

2. (a) The tangents at the extremities of a normal chord of the parabola $x^2=4ay$ meet at a point T. Find the locus of T. 5

   (b) Find the condition to be imposed on k such that the plane $x+kz=1$ intersects the hyperboloid of two sheets $x^2+y^2-z^2+1=0$ in an ellipse. 5

3. (a) Find the locus of a luminous point if the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ casts a circular shadow on the plane $x=0$. 5

   (b) Show that the quadric 5

   $5x^2-4y^2+5z^2+4yz-14zx+4xy+16x+16y-32z+8=0$ represents two intersecting planes.

Group-B

Answer any ***one*** question from the following — 10×1 = 10

4. (a) Find the eigenvalues and eigenfunctions of the boundary value problem 5

   $$y''-4\lambda y'+4\lambda^2 y=0\ ;\ \ y'(1)=0,\ y(2)+2y'(2)=0.$$

(b) Solve $\frac{dx}{dt} - 3x - 6y = t^2$ 5

$\frac{dy}{dt} + \frac{dx}{dt} - 3y = e^t$

5\. (a) Solve $(y^2 + z^2 - x^2)p - 2xyq + 2zx = 0$ by Lagrange's method. 5

(b) Find a complete integral of the equation $xpq + yq^2 = 1$ by Charpit's method. 5

Group-C

Answer either Question No. **6** or Question No. **7** and either Question No. **8** or Question No. **9** 13+12

6\. (a) Prove that every extreme point of the convex set of all feasible solutions of the system $\text{Ax} = \text{b}$, $\mathbf{x} \geq 0$, corresponds to a basic feasible solution. 1+5

(b) Solve the following LPP by Charne's Big M method. 7

Max $Z = 5x_1 - 4x_2 + 3x_3$

Subject to $2x_1 + x_2 - 6x_3 = 20$, $6x_1 + 5x_2 + 10x_3 \leq 76$,

$8x_1 - 3x_2 + 6x_3 \leq 50$, $x_1, x_2, x_3 \geq 0$.

OR

7\. (a) Prove that the number of basic variables in a balanced transportation problem is at most $(m + n - 1)$, where the problem has m origins and n destinations. 5

(b) Find the dual problem of the following primal problem. 8

Min $z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \leq 3$

$2x_2 - x_3 \geq 4$

x_1, $x_2 \geq 0$, x_3 is unrestricted in sign.

8\. (a) Solve the following Transportation problem and find its minimum cost. 7

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

(b) Solve the following 2×2 game (without saddle point) using mixed strategies: 5

B

		B_1	B_2
A	A_1	6	– 4
	A_2	– 1	2

OR

9. (a) Solve the following Travelling salesman problem so as to minimize the cost per cycle.

To

From		A	B	C	D	E
	A	∞	3	6	2	3
	B	3	∞	5	2	3
	C	6	5	∞	6	4
	D	2	2	6	∞	6
	E	3	3	4	6	∞

(b) Solve graphically the following game problem: $\begin{bmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{bmatrix}$. 5

Group-D

Answer any ***three*** questions from the following 15×3 = 45

10.(a) A particle of mass *m* moves on a straight line under a force mn^2x towards a fixed point O on the line, where *x* is the distance from O. If $x = a$ and $\dot{x} = u$ when $t = 0$, show that the amplitude of motion is $\sqrt{a^2 + \frac{u^2}{n^2}}$. 7

(b) Two particles are projected simultaneously from O in different direction with the same speed *u* so as to pass through another point P, if α and β are the angles of projection, prove that they pass through P at times separated by $\frac{2u}{g}\sin\left(\frac{\alpha-\beta}{2}\right)\sec\left(\frac{\alpha+\beta}{2}\right)$. 8

11.(a) Find the radial and cross-radial components of velocity and acceleration of a particle referred to a set of rotating rectangular axes. 8

(b) Find the loss of kinetic energy in direct impact of two smooth elastic spheres of masses *M* and M'. 7

12.(a) A particle describes a plane curve under an acceleration which is always directed towards a fixed point (i.e. under the action of a central force F per unit mass). Find the differential equation of the orbit in polar as well as in pedal form. 8

(b) The curve $x = a(\theta - e\sin\theta)$, $y = a(1 - e\cos\theta)$ where a, e are constants and θ is a parameter, is described by a particle under the action of a force parallel to the axis of x. Show that the force varies as $\dfrac{(e - \cos\theta)}{\sin^3\theta}$. 7

13.(a) State Kepler's laws of planetary motion. Prove that the velocity at the end of the minor axis of a planet's orbit is the geometric mean of the velocities when it is nearest and farthest from the Sun. 8

(b) A particle is projected from the lowest point with velocity $\frac{1}{5}\sqrt{95\,ag}$ along the inner surface of a smooth vertical circle of radius 'a'. Show that it will leave the circle at an angular distance $\cos^{-1}\frac{3}{5}$ from the highest point and that its velocity is then $\frac{1}{5}\sqrt{15ag}$. 7

14.(a) A rough cycloid has its plane vertical and the line joining its cusps horizontal. A heavy particle slides down the curve from rest at a cusp and comes to rest at a point on the other side of the vertex where the tangent is inclined at 45° to the vertical. Show that the coefficient of friction μ satisfies the equation $3\mu\pi + 4\log_e(1+\mu) = 2\log_e 2$. 8

(b) A particle falls from rest under gravity through a stationary cloud. The mass of the particle increases by accumulation from the cloud at the rate of mkv where m is the mass, v is the velocity of the particle at that instant and k is a constant. Show that after the particle has fallen a distance x, its velocity is given by $kv^2 = 1 - e^{-2kx}$. 7