WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-III Examinations, 2018

MATHEMATICS-HONOURS

PAPER- MTMA-V

Time Allotted: 4 Hours — Full Marks: 100

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Group-A
(Marks: 70)

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: $3\times 5 = 15$

(a) Correct or justify: If $S=(-1,\ 1)$ and $T=\{n\,|\,n\in\mathbb{Z}$ and $-m\le n\le m$, for some fixed integer $m>0\}$ then $S\cup T$ is compact.

(b) If $f:[a,\ b]\to\mathbb{R}$ be continuous in $[a,\ b]$, $f(x)>0$ and

$F(x)=\int\limits_a^x f(t)dt,\ a\le x\le b$; then prove that F is strictly increasing in $[a\ ,\ b]$.

(c) Show that the arc of the upper half of the cardioide $r=a(1-\cos\theta)$ is bisected at $\theta=2\pi/3$.

(d) Find the radius of convergence of the power series $x+\sum\limits_{n=2}^{\infty}\frac{(n!)^2}{(2n)!}x^n$.

(e) Examine the convergence of $\int\limits_0^2\frac{\log x}{\sqrt{2-x}}dx$.

(f) Show that function $f(x)=|x-1|$ is a function of bounded variation on $[0,\ 2]$.

(g) If $\log x=\int\limits_1^x\frac{1}{t}dt, x>0,$ show that $\log x$ is strictly increasing on $(0,\ \infty)$ and $\lim\limits_{x\to\infty}\log x=\infty$.

(h) Show that $\sum\limits_{n=1}^{\infty}\frac{(n+1)^3x^n}{3^n\cdot n^5}$ is uniformly convergent on $[–3, 3]$.

(i) Show that $\iint\limits_E(x^2+y^2)\ dxdy=\frac{6}{35}$, where E is the region bounded by $y=x^2$ and $y^2=x$.

2. (a) Prove that a compact subset of $\mathbb{R}$ is closed and bounded in $\mathbb{R}$. 4

   (b) If $f : D \to \mathbb{R}$ be a continuous function on a compact subset D of $\mathbb{R}$ then prove that $f(D)$ is compact in $\mathbb{R}$. 4

   (c) If $G = \left\{ \left(\dfrac{x}{2}, \dfrac{1}{2}(x+1) \right) : x \in (0, 1) \right\}$ then show that G is an open cover of (0, 1) but it has no finite sub cover for (0, 1). 3

3. (a) If $f : [a, b] \to \mathbb{R}$ be bounded on $[a, b]$ then prove that f is Riemann integrable on $[a, b]$ if and only if for $\varepsilon > 0$, there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$. 4

   (b) A function f is defined on [0, 1] by 4

$$f(x) = \begin{cases} \sin x, & x \text{ is rational} \\ x, & x \text{ is irrational} \end{cases}$$

   Evaluate $\overline{\int_0^{\pi/2}} f$ and $\underline{\int_0^{\pi/2}} f$ and hence examine the integrability of f on [0, 1].

   (c) Prove that $\left| \int_a^b \dfrac{\cos x}{1+x} dx \right| < \dfrac{4}{1+a}$, $0 < a < b$. 3

4. (a) Let a be the only point of infinite discontinuity of the functions f and g which are both integrable on $[a+\varepsilon, b]$ for all ε satisfying $0 < \varepsilon < b - a$ and $f(x) > 0, g(x) > 0, \forall\, x \in (a, b]$. 4

   If $\lim_{x \to a+} \dfrac{f(x)}{g(x)} = l$, where l is a non-zero finite number, then prove that $\int_a^b f(x)dx$ and $\int_a^b g(x)dx$ converge or diverge together.

   (b) Find the value of α for which $\int_0^\infty \dfrac{x^{\alpha-1} \log x}{1+x} dx$ will converge. 4

   (c) Using Dirichlet's test, show that $\sum_{n=1}^{\infty} \dfrac{\cos nx}{n}$ is a uniformly convergent series on any closed interval $[a, b]$ contained in $(0, 2\pi)$. 3

5. (a) If for each $n \in \mathbb{N}$, $f_n : [a, b] \to \mathbb{R}$ be a function such that $f_n'(x)$ exists for all $x \in [a, b]$; $\{f_n(c)\}_n$ converges for some $c \in [a, b]$ and the sequence $\{f_n'\}_n$ converges uniformly then prove that the sequence $\{f_n\}_n$ converges uniformly on $[a, b]$. 4

(b) For $n \in \mathbb{N}$, $f_n(x) = 4n^2x,\ 0 \le x < \frac{1}{2n}$ 3

$$= -4n^2x + 4n,\ \frac{1}{2n} \le x < \frac{1}{n}$$

$$= 0,\ \frac{1}{n} \le x \le 1$$

Examine if $\{f_n\}_n$ uniformly convergent on [0, 1]?

(c) For each $n \in \mathbb{N}$, $f_n(x) = \dfrac{n^2x}{1+n^4x^2}$, $x \in [0,\ 1]$. Examine uniform convergence of $\{f_n\}_n$. 4

6. (a) For each $n \in \mathbb{N}$, $f_n : D \to \mathbb{R}$ is a continuous function on $D \subset \mathbb{R}$. If the series $\sum f_n$ is uniformly convergent on D then prove that the sum function is continuous on D. 4

(b) Using Abel's test show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n^p(1+x^n)}$ converges uniformly for all $p > 0$ on [0, 1]. 4

(c) Correct or justify : If $\displaystyle\sum_{n=0}^{\infty} |a_n|$ is convergent then $\displaystyle\int_0^1 \left(\sum_{n=0}^{\infty} a_n x^n\right) dx = \sum_{n=0}^{\infty} \frac{a_n}{n+1}$. 3

7. (a) If $\displaystyle\sum_{n=0}^{\infty} a_n x^n$ is a power series with radius of convergence 1 and $\displaystyle\sum_{n=0}^{\infty} a_n$ is convergent then prove that $\displaystyle\sum_{n=0}^{\infty} a_n x^n$ is uniformly convergent on [0, 1]. 5

(b) Show that $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}$ may be integrated term-by-term from 0 to x, $-1 < x < 1$ and thus prove that 4+2

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\cdots (-1 < x < 1).$$

Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \cdots$.

8. (a) If $f : [a,\ b] \to \mathbb{R}$ be Riemann integrable, then prove that 3

$F(x) = \displaystyle\int_a^x f(t)\,dt$ is of bounded variation over [a, b].

(b) A function $f:[0,1]\to\mathbb{R}$ is defined by 3

$$f(x)=\sin\frac{\pi}{x},\ \ 0<x\le 1$$
$$=0,\qquad x=0\,.$$

Is the function f of bounded variation over [0, 1]?

(c) If $f(x)=\{\pi-|x|\}^2$ on $[-\pi,\pi]$, obtain the Fourier series of f. 5

Hence deduce that $\sum\limits_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$.

9. (a) State and prove Mean Value Theorem for a real valued function of two variables. 1+3

(b) Find the expansion of $\cos(xy)$ in powers of $(x-1)$ and $\left(y-\frac{\pi}{2}\right)$ upto and including third degree terms. 3

(c) Using Lagrange's method of multipliers, prove that 4

$$\frac{x^2+y^2+z^2}{3}\ge\left(\frac{x+y+z}{3}\right)^2,\text{ where } x\ge 0,\ y\ge 0,\ z\ge 0\,.$$

10.(a) Evaluate $\int\limits_0^a\frac{\log(1+ax)}{1+x^2}\,dx$, $a>0$ is a parameter. 3

(b) Change the variables in the integral $\int\limits_0^{2a}dx\int\limits_{\sqrt{2ax-x^2}}^{\sqrt{4ax-x^2}}\left(1+\frac{y^2}{x^2}\right)dy$ to r and θ, where $x=r\cos^2\theta$, $y=r\sin\theta\cos\theta$ and show that the value of the integral is $\left(\pi+\frac{8}{3}\right)a^2$. 4

(c) Compute $\iiint\limits_E\frac{dx\,dy\,dz}{x^2+y^2+(z-2)^2}$, where E is the sphere $x^2+y^2+z^2\le 1$. 4

Group-B
[Marks: 15]

Answer any *one* question from the following

11.(a) If (X, d) is a metric space show that the function $\rho: X\times X\to\mathbb{R}$, defined by $\rho(x,y)=\frac{d(x,y)}{1+d(x,y)}$, $x, y\in X$, is also a metric on X. 5

(b) Show that every set in a discrete metric space is an open set as well as a closed set. 4

(c) Define (i) a bounded sequence (ii) a Cauchy sequence in a metric space. Show that in a metric space a Cauchy sequence is bounded. Does the converse hold? Support your answer. 1+1+3+1

12.(a) Let $C[a,b]$ denote the set of all real valued continuous functions defined on the closed interval $[a, b]$. Define $d : C[a,b]\times C[a,b] \to \mathbb{R}$ by 5

$$d(f, g) = \sup_{x\in[a,\, b]} |f(x) - g(x)| \text{ for all } f, g \in C[a, b].$$

Show that d is a metric on $C[a, b]$.

(b) Let (X, d) be a metric space. Show that for any $x, y \in X$, $x \neq y$ there are open balls B_1 and B_2 in (X, d) such that $x \in B_1$, $y \in B_2$, $B_1 \cap B_2 = \Phi$. Also show that $\{x\}$ is closed in $\{X, d\}$ for any $x \in X$. 2+2

(c) Show that in a discrete metric space a convergent sequence is eventually constant. 2

(d) Let (X, d) be a metric space and $A \subset X$. Show that a point $p \in \overline{A}$ if and only if there exists a sequence $\{x_n\}$ in A which converges to p, ($\overline{A}$ denotes the closure of A). 4

Group-C
[Marks: 15]

Answer any *one* question from the following. 15×1 = 15

13.(a) If $f : G \to \mathbb{C}$ is an analytic function defined on a region G in $\mathbb{C}$ and if $|f|$ is constant on G, then show that f is constant on G. 4

(b) Let $\{z_n\}$ be a sequence in $\mathbb{C}$. Let $z_n = x_n + iy_n$ for all $n \in \mathbb{N}$. Then show that $\{z_n\}$ is a convergent sequence in $\mathbb{C}$ if and only if $\{x_n\}$ and $\{y_n\}$ are convergent sequences in $\mathbb{R}$. Furthermore, if $\{z_n\}$ is a convergent sequence in $\mathbb{C}$, show that $\lim_{n\to\infty} z_n = \lim_{n\to\infty} x_n + i \lim_{n\to\infty} y_n$. 4+2

(c) Let $\omega \in \mathbb{C}$. Define $f : \mathbb{C} \to \mathbb{C}$ by $f(z) = |z - \omega|$. Show that f is nowhere differentiable on $\mathbb{C}$. 5

14.(a) If a function $f(x+iy) = u(x, y) + iv(x, y)$ is differentiable at a point $z_0 = x_0 + iy_0$ then show that $u(x, y)$ and $v(x, y)$ are differentiable at (x_0, y_0) and at (x_0, y_0), $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$. 6

Further show that $f'(z_0) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$.

(b) Show that the function 5

$$f(x+iy)=\begin{cases}\dfrac{x^2y^5(x+iy)}{x^4+y^{10}}; & x+iy\neq 0\\ 0; & x+iy=0\end{cases}$$

is not differentiable at the origin even though it satisfies Cauchy-Riemann equations there.

(c) Show that $u(x, y)=e^x\cos y$ is a harmonic function. Determine a conjugate harmonic function of *u*. 4

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Mathematics-Honours

Paper- MTMA-VI

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Group-A

Answer any *two* questions from Question No. 1 to 3 and any *one* from Question No. 4 and 5

1. Answer any ***three*** questions from the following: 5×3 = 15

(a) What is meant by the term 'statistical regularity'? Explain how the frequency definition of the probability of a random event related to the concept of statistical regularity. Starting from the frequency definition of probability establish the following: 1+1+3

$P(A_1 + A_2 + \ldots + A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$, where $A_i's$ are mutually exclusive random events for $i = 1, 2, \ldots, n$.

(b) If a die is thrown k times, show that the probability of even number of sixes is $\frac{\{1+(2/3)^k\}}{2}$. 5

(c) Let A, B, C be mutually independent events. Then prove that A and $B + C$ are independent and also that $\overline{A}$, $\overline{B}$, $\overline{C}$ are mutually independent. 5

(d) A missile has probability $\frac{1}{2}$ of destroying it target and probability $\frac{1}{2}$ of missing it. Assuming that the missile firings form independent trials, determine the least number of missiles that should be fired at a target in order to make the probability of destroying the target at least 0.99. 5

(e) If A_n be a monotone sequence of random events, then prove that 5

$$P(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} P(A_n).$$

2. Answer any ***three*** questions from the following: 5×3 = 15

(a) If a Random Variable $X \sim B(n, p)$, prove that $\mu_{k+1} = p(1-p)(nk\mu_{k-1} + \frac{d\mu_k}{dp})$, where μ_k is the k-th central moment. Hence obtain the mean and variance of X. 5

(b) In the equation $x^2 + 2x - q = 0$, q is a random variable uniformly distributed over the interval (0, 2). Find the distribution function of the larger root. 5

(c) The joint density function of the random variables X, Y is given by 5

$$f(x, y) = K(3x + y), \text{ when } 1 \le x \le 3 \text{ and } 0 \le y \le 2,$$
$$= 0, \quad \text{Otherwise}$$

Find: (i) $P(X + Y < 2)$ (ii) The marginal distribution of X and Y. Investigate whether X and Y are independent.

(d) X is a continuous random variable having a probability density function (p.d.f) $f_X(x)$. Let y be a continuously differentiable function of x. Show that the p.d.f $f_Y(y)$ of random variable $Y = g(X)$ is given by $f_y(y) = f_x(x)\left|\frac{dx}{dy}\right|$ 5

(e) If X follows standard normal distribution, show that $\frac{1}{2}X^2$ follows gamma distribution with parameter $\frac{1}{2}$. 5

3. Answer any ***three*** questions from the following: 5×3 = 15

(a) A point P is chosen at random on a line segment AB of length $2l$. Find the expected values of 5

(i) $AP.PB$ (ii) $|AP - PB|$

(b) Let (X, Y) be a two-dimensional random variable. Prove that $[E(XY)]^2 \le E(X^2)E(Y^2)$. Use this result to prove that $-1 \le \rho \le 1$, where ρ is the correlation coefficient between X and Y. 5

(c) Define concept of convergence in probability. Let $X_n \xrightarrow[\text{in } p]{} a$ as $n \to \infty$ and $Y_n \xrightarrow[\text{in } p]{} b$ as $n \to \infty$, then show that $X_nY_n \xrightarrow[\text{in } p]{} ab$ as $n \to \infty$. 5

(d) The random variables X, Y are both standard normal and are mutually independent. Find the expectation of max $\{|X|, |Y|\}$. 5

(e) If X_n be Binomial (n, p) variate, then show that $\frac{X_n - np}{\sqrt{npq}}, (q = 1 - p)$ is asymptotically Normal (0, 1). 5

4. (a) Define an unbiased and consistent estimate of a parameter connected with the distribution function of a population. Prove that sample mean is always unbiased and consistent estimate of the population mean. 2+4

(b) Find the maximum likelihood estimate of the parameter α of the continuous population having the density function $f(x) = (1+\alpha)x^{\alpha}, 0 < x < 1$ where $\alpha > -1$. 6

(c) What is meant by a statistical hypothesis? 2+6

A drug is given to 10 patients and the increment of blood pressure were recorded to be 3, 6, – 2, 4, – 3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on change of blood pressure? It may be assumed that for 9 degrees of freedom $P(t > 2.262) = 0.025$.

5. (a) Obtain a test for null hypothesis $H_0 : m = m_0$ against the alternative hypothesis $H_1 : m < m_0$ for a normal (m, σ) population when σ is known. 6

(b) Two random variables X and Y are connected by the relation $3X + 4Y + 5 = 0$. A sample (x_i, y_i), $i = 1, 2, \ldots n$ is taken from the bivariate population of (X, Y); obtain the correlation coefficient of the sample. 6

(c) What do you mean by confidence interval for a parameter of a distribution? Find a confidence interval for mean of normal (m, σ) population when (i) σ is known (ii) σ is unknown. 1+4+3

Group-B
Section-I
[Marks: 30]

Answer any *three* questions from the following 10×3 = 30

6. (a) What are the different sources of computational errors in a numerical computational work? Discuss with suitable examples. 2

(b) Find the relative percentage error in $f(x)$ for $x = 0$, if the error in x is 0.002, where $f(x) = x^2 - 6x + \sin x$. 2

(c) Define a confluent divided difference of order one. 2

(d) Write down the remainder term associated with Newton's forward interpolation formula with $(n+1)$ equispaced interpolating points $x_0, x_1, \ldots, x_n$. Hence show that the maximum absolute error in linear interpolation is given by $h^2 M_2 / 8$ where M_2 is $\max\limits_{x_0 \le x \le x_1} |f''(x)|$. 4

7. (a) Explain the principle of numerical differentiation. Deduce Lagrange's differentiation formulae, both of 1st and 2nd order (without error term). 1+4

(b) Describe Hermit interpolation. Deduce the interpolation formula. 5

8. (a) State the general principle of Newton-Cotes' closed type formula for evaluating an integral of the form $\int_a^b f(x)dx$ where a, b are finite. Hence or otherwise obtain the trapezoidal rule. 4+1

(b) Describe Gauss' Elimination method for numerical solution of a system of linear equations and explain the pivoting process in this connection. 5

9. (a) Describe Bisection method for computing a simple real root of $f(x)=0$. Give a geometrical interpretation of the method and also the error estimate. 5

(b) Solve the equation $\frac{dy}{dx}=\frac{1}{x+y}$, $y(0)=1$ by modified Euler's method to obtain $y(0.2)$ and $y(0.4)$ correct up to 4D. 5

10.(a) Explain the method of Regula-Falsi for computing a real root of an equation $f(x)=0$ and also explain the geometrical interpretation of the process. 5

(b) Solve the equation $\frac{dy}{dx}=x^2+y^2$, $y(0)=1$ by fourth order Runge Kutta method for $x=\text{O}(0.1)\,0.2$ correct up to 4D. 5

Section-II

[Marks: 20]

Answer any *two* questions from the following 10×2 = 20

11.(a) Discuss primary memory and secondary memory. What is the fundamental unit of measuring memory? 3+2

(b) Draw a flowchart to find n! (n is a positive integer).

(c) (i) Convert $(520.375)_{10}$ into octal form. 2+2+1

(ii) Use 2's complement to compute $(1110.1001)_2 - (1010.011)_2$

(iii) Find the CNF of $xy+x'y$.

12.(a) Write a FORTRAN 77/90 or C program to input 5 numbers and print the biggest of the five. 5

(b) Write a FORTRAN 77/90 or C program to evaluate 5

$$e=1+\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{100}+\cdots \text{ correct to 4D.}$$

13.(a) Write a program in FORTRAN 77/90 or C to find a real root of the equation $x^2e^{-x}-x+0.2=0$ by the methods of iteration correct up to 6 decimal places. 5

(b) Write a FORTRAN 77/90 or C program to test whether a given number is divisible by 7 but not by 3. 5

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Paper- MTMA-VII

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Group-A

(VECTOR ANALYSIS-II)

Answer any *one* question from the following 10×1 = 10

1. (a) Prove that $\iiint_V \frac{dV}{r^2} = \iint_S \frac{\vec{r}\cdot\vec{n}}{r^2}\,dS$ where S is any closed surface enclosing a volume V. 5

(b) If $\vec{F} = (3x^2+6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, then evaluate $\int_C \vec{F}\cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path given by $x=t$, $y=t^2$, $z=t^3$. 5

2. (a) Using Stokes' theorem, prove that $\text{div}(\text{curl}\,\vec{F}) = 0$ and $\text{curl}(\text{grad}\,\phi) = \vec{0}$. 5

(b) Prove that the necessary and sufficient condition that $\oint \vec{F}\cdot d\vec{r} = 0$ for every closed curve C is that $\vec{\nabla}\times\vec{F} = \vec{0}$ identically. 5

Group-B

(ANALYTICAL STATICS)

Answer any *five* questions from the following 7×5 = 35

3. Explain the terms 'force of friction' and angle of friction. A uniform ladder with its lower end on a rough ground leans against a smooth vertical wall. Prove that

(i) If the inclination θ of the ladder to the wall is less than the angle of friction λ, no load placed on the ladder, however large, can make it slip;

(ii) If $\lambda < \theta < \tan^{-1}(2\tan\lambda)$, the ladder can be made slip by placing on it an additional load, and

(iii) If $\theta > \tan^{-1}(2\tan\lambda)$ the ladder will slip without any additional load.

4. Prove that if all the forces of a coplanar system acting on a rigid body are rotated about their point of application through the same angle in their plane, their resultant passes through a fixed point in the body.

5. Three forces act along the straight lines $x=0$, $y-z=a$; $y=0$, $z-x=a$; $z=0$, $x-y=a$. Show that they can not reduce to a couple. Prove also that if the system reduces to a single force its line of action must lie on the surface $x^2+y^2+z^2-2yz-2zx-2xy=a^2$.

6. Find the centre of gravity of the area of the cardioid $r=a(1+\cos\theta)$.

7. Determine the conditions of equilibrium of a particle constrained to rest on a rough plane curve $y=\phi(x)$ under the action of any given forces.

8. What is the energy test of stability? Establish the energy test of stability for a rigid body with one degree of freedom only, in equilibrium under conservative forces.

9. A body rests in equilibrium on another fixed body having enough friction to prevent sliding, the portion of the two bodies in contact are spherical and of radii r and R respectively and the line joining their centers in position of equilibrium is vertical. Show that the equilibrium is stable if $\frac{1}{h}>\frac{1}{r}+\frac{1}{R}$ where h is the height of the C.G. of the body in position of equilibrium above the point of contact.

10. Two uniform rods *AB*, *BC* of weight W and W' are smoothly jointed at *B* and their middle points are joined across by a chord. The rods are tightly held in a vertical plane with their ends *A*, *C* resting on a smooth horizontal plane. Show by using the principle of virtual work that the tension in the chord is $(W+W')\cos A\cdot\cos C/\sin B$.

11. A force P acts along the axis of x and another force nP acts along a generator of the cylinder $x^2+y^2=a^2$. Show that the central axis lies on the cylinder $n^2(nx-z)^2+(1+n^2)^2y^2=a^2n^4$.

Group-C
(RIGID DYNAMICS)

Answer any *two* questions from the following. 15×2 = 30

12.(a) Find whether a given straight line is, at any point of its length, a principal axis of a material system. If it is so, then find the other two principal axes at that point. Hence show that if an axis passes through the centre of inertia of a body and is a principal axis at some point of its length, then it is a principal axis at all points of its length. 8

(b) Prove that the moment of inertia of a triangular lamina ABC about a perpendicular to the plane through the vertex A is $\frac{M}{3}(3b^2+3c^2-a^2)$, where *a*, *b*, *c* are lengths of the sides of the triangle and *M* is its mass. 7

13.(a) A compound pendulum of mass M oscillating about a fixed horizontal axis has its centre of oscillation at C. Find the period of oscillation of the compound pendulum. Show further that the period is unaltered even if a weight is rigidly attached to the body of the pendulum at C. 8

(b) Find the moment of momentum of a rigid body moving in two dimensions about the origin. 7

14.(a) An imperfectly rough sphere moves from rest down a plane inclined at an angle α to the horizon. Determine the conditions on μ for (i) never rolling (ii) rolling and no sliding (iii) pure rolling from the start. 8

(b) A uniform rod AB is freely movable on a rough inclined plane, whose inclination to the horizon is α and whose coefficient of friction is μ, about a smooth pin fixed through the end A; the rod is held in the horizontal position in the plane and allowed to fall from this position. If θ be the angle through which it falls from rest, show that $\frac{\sin\theta}{\theta}=\mu\cot\alpha$. 7

Group-D

(HYDROSTATICS)

Answer any *two* questions taking *one* question from each section.

SECTION-I

15×1 = 15

15.(a) A mass of liquid is in equilibrium under the action of conservative system of forces. Show that the surface of equi-pressure, equi density, and equi-potential energy coincide. If the system of forces is the force of gravity only, show that these surfaces are horizontal. 8

(b) A given volume V of a heavy liquid is acted on by forces $-\mu x$, $-\mu y$, $-\mu z$. Find the equation of the free surface. 7

16.(a) Prove that the depth of centre of pressure of a plane area immersed in a liquid under gravity is greater than that of the centre of mass of the area. What happens when the area is lowered further? 8

(b) If a floating solid be a cylinder, with its axis vertical, the ratio of whose specific gravity to that of the fluid is σ, prove that the equilibrium will be stable, if the radius of the base to the height is greater than $[2\sigma(1-\sigma)]^{\frac{1}{2}}$. 7

SECTION-II

10×1 = 10

17.(a) A gass satisfying Boyle's law $p = k\rho$ is acted on by forces $X = \dfrac{-y}{x^2 + y^2}$, $X = \dfrac{x}{x^2 + y^2}$. Show that the density varies as $e^{\frac{\theta}{k}}$ where $\tan\theta = \dfrac{y}{x}$. 5

(b) A cone whose vertical angle is 2α, has the lowest generator horizontal and is filled with liquid. Prove that the resultant thrust on the curved surface is $\sqrt{1+15\sin^2\alpha}$ times the weight of the liquid. Also determine the inclination of the thrust. 5

18.(a) Find the condition for existence of metacentre of a body and prove the formula $HM = \dfrac{AK^2}{V}$, with usual notations, for finding the metacentre of the body floating freely in a homogeneous liquid at rest under gravity. 5

(b) Find the thrust on a vertical quadrilateral which has one side of length *a* in the surface and the opposite side of length *b* parallel to it at depth *h*. 5

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-III Examinations, 2018

MATHEMATICS-HONOURS

PAPER- MTMA-VIII-A

Time Allotted: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Group-A

Section-I

LINEAR ALGEBRA

Answer any *one* question from the following $10\times1 = 10$

1. (a) If V and W are two finite dimensional vector spaces and $\mathrm{T}:\mathrm{V}\to\mathrm{W}$ is a one-one linear map, then show that the images of linearly independent set of vectors in V are linearly independent in W. 2

(b) Is there a linear transformation $T:\mathbb{R}^3\to\mathbb{R}^2$ such that $T(1,-1,1)=(1,0)$ and $T(1,1,1)=(0,1)$? Justify. 3

(c) For a positive integer n, P_n denotes the vector space of polynomials of degree $\le n$, over the field of real numbers. Determine explicitly the linear map $T:P_3\to P_2$ which maps the vectors $1,\ 1-x,\ x^2,\ x^3$ in P_3 respectively to the vectors $x^2,\ 0,\ x,\ 1+x$ in P_2. Then calculate the matrix of T relative to the ordered bases $B_1=\{1,\ x-1,\ (x-1)^2,\ (x-1)^3\}$ and $B_2=\{1,\ x,\ x^2\}$. 3+2

2. (a) Let $T:\mathrm{V}\to\mathrm{U}$ and $S:\mathrm{U}\to\mathrm{W}$ be linear maps where, V, U, W are finite dimensional vector spaces over a field F. Then relative to a choice of ordered bases, show that $m(S\cdot T)=m(S)\cdot m(T)$ [where $m(T)$ stands for the matrix of T with respect to the chosen basis]. 5

(b) For a positive integer n, P_n denotes the vector space of polynomials of degree $\le n$, over the field of real numbers. Let $T:P_2\to P_3$ be a linear transformation defined by 3

$$T\big(f(x)\big)=2f'(x)+\int_0^x 3f(t)\,dt\text{, for all } f(x)\in P_2.$$

Prove that T is injective.

(c) The matrix representation of a linear mapping $T:\mathbb{R}^3 \to \mathbb{R}^3$ is $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ relative to the standard basis of $\mathbb{R}^3$. Find the explicit representation of *T*. 2

Section-II

MODERN ALGEBRA

Answer any *one* question from the following 8×1 = 8

3. (a) Define kernel of a homomorphism with example. 2

(b) Prove that if $\phi:(G,\circ)\to(G',*)$ be a homomorphism then $\ker\phi$ is a normal subgroup of *G*. 2

(c) Show that two finite cyclic groups are isomorphic. 4

4. (a) If $(H,\circ)$ is a normal subgroup of a group $(G,\circ)$, then prove that the quotient group $(G/H\,,*)$ is Abelian if and only if 4

$$x\circ y\circ x^{-1}\circ y^{-1}\in H \quad \forall x,\, y\in G\,.$$

(b) If *G* is a commutative group and $\phi:G\to G'$ is an epimorphism from *G* to any group G', then show that G' is also commutative. Is the converse true? Justify. 2+2

Section-III

BOOLEAN ALGEBRA

Answer any *one* question from the following 7×1 = 7

5. (a) Define a Boolean Algebra. Prove that $P(A)$, the power set of a non-empty set *A*, forms a Boolean Algebra with respect to the set union, intersection and complementation. 1+3

(b) Draw the circuit that realises the function *f* given in the following table: 3

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

6. (a) Express the Boolean expression $(x+y)(x+y')(x'+z)$ in DNF in the variable *x*, *z* and also express it in DNF in the variables *x*, *y*, *z*. 4

(b) If *x*, *y*, *z* are three switches, then draw a switching circuit representing $[zx+x(y+z')]\,(z+x)(z+y)$. 3

Group-B

DIFFERENTIAL EQUATIONS-III

Answer any *one* question from the following 15×1 = 15

7. (a) Obtain the series solution of $(1-x^2)\dfrac{d^2y}{dx^2}+2y=0$, given $y(0)=4$, $y'(0)=5$. 5

(b) Using change of scale property, evaluate $\mathrm{L}\{3\cos 6t-5\sin 6t\}$ and hence show that $\mathrm{L}\{e^{-2t}(3\cos 6t-5\sin 6t)\}=\dfrac{3(s-8)}{s^2+4s+40}$. 5

(c) Solve using Laplace transform: 5

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{-2x}\sin x,\ y(0)=0,\ y'(0)=0.$$

8. (a) Applying power series method, solve $\dfrac{d^2y}{dx^2}-y=x$. 5

(b) If $F(s)=\dfrac{1}{(s^2+a^2)(s^2+b^2)}(a\neq b)$, then find $f(t)$, where $f(t)=\mathrm{L}^{-1}\{F(s)\}$. 5

(c) Using first shifting property of Laplace transform evaluate 5

$$\mathrm{L}^{-1}\left(\frac{s-10}{s^2-4s+20}\right).$$

Group-C

TENSOR CALCULUS

Answer any *one* question from the following 10×1 = 10

9. (a) If A^i and B^i are two non-null vectors such that $g_{ij}U^iU^j=g_{ij}V^iV^j$, where $U^i=A^i+B^i$ and $V^i=A^i-B^i$. Show that A^i and B^i are orthogonal. 2

(b) Prove that sum of two tensors of same type is a tensor of same type. 2

(c) Prove that Christofel symbols are not tensors. 3

(d) Show that in a Riemannian space V_n of dimension *n*, with metric tensor g_{ij}, 3

$\begin{Bmatrix} i \\ i\ j \end{Bmatrix} = \frac{\partial}{\partial x^j}\left(\log\sqrt{g}\right)$, where $g = |\, g_{ij}\, |$.

10.(a) Line element of two neighboring points $P(x^i)$ and $Q(x^i + dx^i)$ in a 3-dimensional space is given by 4

$$ds^2 = (dx^1)^2 + 2(dx^2)^2 + 3(dx^3)^2 - 2dx^1dx^2 + 4dx^2dx^3 .$$

By this line element, does the above space form a Riemannian space? Justify it.

(b) Show that $g_{ik,\,j} = 0$ and $\delta^i_{k,\,j} = 0$. 2+1

(c) Show that $g_{ij}\, dx^i dx^j$ is an invariant, where g_{ij} is the fundamental metric tensor. 3