WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Examination, 2021-22

MTMACOR07T-MATHEMATICS (CC7)

Time Allotted: 2 Hours   Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *four* questions from the rest

1.   Answer any *four* questions from the following: 2×4 = 8

(a) Given $f(0) = 3$, $f(1) = 12$, $f(2) = 81$, $f(3) = 200$, $f(4) = 100$, $f(5) = 8$. Find $\Delta^5 f(0)$.

(b) If $u = xyz^2$ and errors in x, y, z are 0.005, 0.002, 0.001 respectively at $x = 3$, $y = z = 1$, compute the maximum absolute error in evaluating u.

(c) Show that $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$.

(d) State Newton-Gregory's backward interpolation formula with its remainder term.

(e) Find an iterative formula to obtain the cube root of a positive number N.

(f) Find the solution of the differential equation

$$\frac{dy}{dx} = 1 + y \quad, \quad y(0) = 0$$

for $x = 0.2$ by using Euler's method (take step length $h = 0.1$).

(g) Find the value of $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx$ by Trapezoidal rule.

2. (a) If $u = 4x^2 y^3 / z^4$ and error in x, y, z be 0.001, compute the relative maximum error in u when $x = y = z = 1$.   4

   (b) Find $y(3)$ from the following data: 4

$$y(0) = 1, \quad y(1) = 3, \quad y(2) = 9, \quad y(4) = 81$$

3. (a) Find a real root of $x^3 - x = 1$ lying between 1 and 2 by Bisection method. Compute 6 iterations.   3

   (b) Write down the geometrical interpretation of the Newton-Raphson method.   2

   (c) Derive the convergence condition for Newton-Raphson method.   3

4. (a) Derive Newton's Backward interpretation formula. 4

   (b) Use Stirling's formula to find $y(28)$, given that 4

   $y(20) = 49225$, $y(25) = 48316$, $y(30) = 47236$, $y(35) = 45926$, $y(40) = 44306$

5. Find the inverse of the matrix 8

   $$A = \begin{pmatrix} 5 & 2 & 1 \\ 2 & 1 & 3 \\ 3 & 3 & 2 \end{pmatrix}$$

   using LU decomposition method and hence solve the system of equations

   $$5x + 2y + z = 12$$
   $$2x + y + 3z = 13$$
   $$3x + 3y + 2z = 15$$

6. (a) A train is moving at the speed of 30 km/sec. Suddenly breaks are applied. The speed v of the train per second after t seconds is given by: 4

Time (t)	0	5	10	15	20	25	30	35	40	45
Speed (v)	30	24	19	16	13	11	10	8	7	5

   Apply Simpson's $3/8^{th}$ rule to determine the distance moved by the train in 45 seconds.

   (b) Write an algorithm to find the sum of only even numbers out of first N numbers input by the user. 4

7. (a) Apply Euler's method to the initial value problem $\frac{dy}{dx} = x + y$, $y = 0$ when $x = 0$, at $x = 0$ to $x = 1.0$ taking $h = 0.2$. 4

   (b) Deduce numerical differentiation formula based on Lagrange's interpolation formula. 4

8. (a) Compute the values of the unknown in the system of equations by Gauss-Jordan method. 5

   $$x_1 + 3x_2 + 2x_3 = 17$$
   $$x_1 + 2x_2 + 3x_3 = 16$$
   $$2x_1 - x_2 + 4x_3 = 13$$

   (b) Prove that the n-th divided difference can be expressed as the quotient of two determinants of order $(n+1)$. 3

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