WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2021-22

MTMACOR11T-MATHEMATICS (CC11)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Form the partial differential equation by eliminating arbitrary function from the following relation:

$$z=\phi(x^2+y^2)$$

(b) Solve the following partial differential equation:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$$

(c) Classify the partial differential equation (elliptic, parabolic or hyperbolic)

$$3\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+3\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}=0$$

(d) State the two dimensional Laplace's equation in cartesian coordinates.

(e) Find the degree and order of the partial differential equations:

(i) $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}+x^2\left(\frac{\partial z}{\partial y}\right)^3=xy^2z^2$ (ii) $\left(1+\frac{\partial^2 z}{\partial y^2}\right)^2=k\left(\frac{\partial z}{\partial x}\right)^4$

(f) If ω be the angular velocity of a planet at the nearer end of the major axis, prove that its period is $\frac{2\pi}{\omega}\sqrt{\frac{1+e}{(1-e)^3}}$

(g) A particle describes a circle about the centre of force which is the centre of the circle. Find the velocity of the particle at any point on the orbit.

(h) Write down the equation of motion of a spherical raindrop falling freely increases at each instant by an amount equal to μ times its surface area at that instant, where a is the initial radius of the raindrop.

(i) A particle describes the curve $p^2=ar$ under a force F to the pole. Find the law of force.

2. (a) Find the differential equation of all spheres of radius a, having centre in the xy-plane. 3

(b) Find the equation of the integral surface given by the equation $2y(z-3)p+(2x-z)q=y(2x-3)$, which passes through the circle $z=0$, $x^2+y^2=2x$. 5

3. (a) Reduce the equation $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=z$ to its canonical form and hence find its general solution. 5

(b) Solve: $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=z^2$. 3

4. Solve the following one dimensional heat equation: 8

$$\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$$

Subject to the conditions $u(0,t)=0$, $u(l,t)=0$ and $u(x,0)=\sin\left(\frac{\pi x}{l}\right)$.

5. Solve the equation by method of separation of variables: 8

$$\frac{\partial^2 u}{\partial x^2}+2\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$$

6. Show that the solution of the following Cauchy problem is unique 8

$$\frac{\partial^2 u}{\partial t^2}-c^2\frac{\partial^2 u}{\partial x^2}=F(x,t),\ 0<x<l,\ t>0$$

subject to the initial conditions $u(x,0)=f(x)$, $0\le x\le l$, $u_t(x,0)=g(x)$, $0\le x\le l$ and the boundary conditions $u(0,t)=u(l,t)=0,\ t\ge 0$.

7. Solve the following Laplace's equation at any interior point to the rectangle formed by $0\le x\le\pi$, $0\le y\le\pi$; $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$ subject to the boundary conditions $\phi_x(0,y)=\phi_x(\pi,y)=\phi_y(x,0)=0$ and $\phi_y(x,\pi)=f(x)$, a function of $x\in[0,\pi]$. 8

8. A particle moves with a central acceleration $\mu\left(r+\frac{2a^3}{r^2}\right)$ being projected from an apse at a distance 'a' with a velocity which is twice the velocity for a circle at that distance. Find the other apsidal distance and show that the equation of the path is 8

$$\frac{\theta}{2}=\tan^{-1}(z\sqrt{3})-\frac{1}{\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{5}{3}}z\right),\ \text{where } z^2=\frac{r-a}{3a-r}$$

9. A particle slides from rest from a cusp down the arc of a rough cycloid whose axis is vertical and vertex downwards. Prove that its velocity at the vertex is to its velocity at the same point when the cycloid is smooth is $(e^{-\mu\pi}-\mu^2)^{1/2}:(1+\mu^2)^{1/2}$, where μ is the coefficient of friction. 8

10. A particle of mass M is at rest and begins to move under the action of a constant force F in a fixed direction. It encounters the resistance of a stream of fine dust moving in the opposite direction with velocity V, which deposits matter on it at a constant rate ρ, show that its mass be m when it has travelled a distance $\frac{k}{\rho^2}\left[m-M\left\{1+\log\left(\frac{m}{M}\right)\right\}\right]$, where $k=F-\rho V$. 8

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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