WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2021

MTMACOR09T-MATHEMATICS (CC9)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5=10

   (a) If S be the set of all points (x, y, z) in 3-space satisfying the inequality $x+y+z<1$, determine whether or not S is open. 2

   (b) Is the set $\mathbb{R}^n$ open? — Justify. 1+1

   (c) Find the closure of $\{(x, y) : 1<x^2+y^2<2\}$. 2

   (d) When a rational function $f(x)=\frac{P(x)}{Q(x)}$ (where P, Q are polynomials in the components of x) is continuous at each point x ? 2

   (e) State a sufficient condition for differentiability of a function in $\mathbb{R}^2$. 2

   (f) Find the gradient vector at each point at which it exists for the scalar field defined by $f(x, y)=x^2+y^2\sin(xy)$. 2

   (g) Prove that every continuous function is double integrable. 2

   (h) Express the concept of work done as a line integral. 2

   (i) Use Green's theorem to compute the work done by the force field $f(x, y)=(y+3x)i+(2y-x)j$ in moving a particle once around the ellipse $4x^2+y^2=4$ in the counterclockwise. 2

2. (a) If $f(x, y)=(x^2+y^2)\log(x^2+y^2)$, when $x^2+y^2\neq 0$
   $= \quad 0$, when $x^2+y^2=0$
   Show that $f_{xy}(0, 0)=f_{yx}(0, 0)$ although neither $f_{xy}(x, y)$ nor $f_{yx}(x, y)$ is continuous at (0, 0). 4

   (b) Show that the function is discontinuous at (0, 0), 4

$$f(x, y)=\begin{cases}\dfrac{x^3+y^3}{x-y} & , \quad \text{when } x\neq y \\ 0 & , \quad x=y\end{cases}$$

3. (a) Prove that the function 4

$$f(a, b)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}} & \text{if } (x, y)\neq(0, 0) \\ 0 & \text{if } (x, y)=(0, 0)\end{cases}$$

   is continuous at (0, 0).

(b) Define closure of a set in $\mathbb{R}^2$. Find the closure of $\{(x, y) : x^2 + y^2 < 1\}$. 4

4. (a) Show that $A \times B$ in $\mathbb{R}^2$ is closed whenever A, B are so in $\mathbb{R}^2$. 4

(b) If $z = x^2 + 2xy$ then prove that dz at the point (1, 1) can be expressed as $dz = 4dx + 2dy$. 4

5. (a) Find $\frac{du}{dt}$ if $u = x^3 - y \sin xy$ and $x = \frac{(t-1)}{t}$, $y = t \cos t$. 4

(b) Find the directional derivative of $f(x, y) = 2x^2 - xy + 5$ at (1, 1) in the direction of unit vector $\beta = \frac{1}{5}(3, 4)$. 4

6. (a) Using the transformation $x + y = u$, $y = uv$, find the value of integral 4

$$\int_0^1 \int_{y=0}^{1-x} e^{\frac{y}{x+y}} \, dy \, dx$$

(b) Evaluate the integral $\iint \frac{dxdy}{(1+x^2+y^2)^2}$ taken over the region of one loop of the lemniscate $(x^2 + y^2)^2 - (x^2 - y^2) = 0$. 4

7. Evaluate $\iint_E f(x, y) \, dx \, dy$ over the rectangle $R = [0, 1; 0, 1]$, where 8

$$f(x, y) = \begin{cases} x + y & \text{if } x^2 < y < 2x^2 \\ 0 \ , & \text{otherwise} \end{cases}$$

8. (a) Show that the vector field given by $A = (y + \sin z, x, x \cos z)$ is conservative. Find the scalar point function for the field. 4

(b) Evaluate $\int_C (\sin z \, dx - \cos x \, dy + \sin y \, dz)$ by Stokes Theorem, where C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$. 4

9. (a) Evaluate the line integral $\int_C [(x^2 - 2xy)dx + (x^2 y + 3)dy]$ by using Green's theorem, around the boundary C of the region defined by $y^2 = 8x$, $x = 2$. 4

(b) Find the work done of a particle in the force field $\mathbf{F} = (2x - y + 4z, x + y - z^2, 3x - 2y + 4z^3)$ moving round the circle $x^2 + y^2 = 4$, $z = 0$. 4

10. Find the volume enclosed by the surfaces $x^2 + y^2 = cz$, $x^2 + y^2 = 2ax$, $z = 0$. 8

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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