WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2021-22

MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours   Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1.   Answer any *five* questions from the following: 2×5 = 10

(a) If $y = \sin kx + \cos kx$, prove that $y_n = k^n \{1 + (-1)^n \sin 2kx\}^{1/2}$.

(b) Find the asymptotes of the curve $x = \dfrac{t^2}{1+t^3}$, $y = \dfrac{t^2+2}{t+1}$.

(c) Determine a such that, $\displaystyle\lim_{x\to 0} \dfrac{a\sin x - \sin 2x}{\tan^3 x}$ exists and $= 1$.

(d) Determine the angle of rotation of the axes so that the equation $x + y + 2 = 0$ may reduce to the form $ax + b = 0$.

(e) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z = 71$.

(f) Find the values of a for which the plane $x + y + z = a\sqrt{3}$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$.

(g) Find the equation of the cylinder whose generating line is parallel to z-axis and the guiding curve is $x^2 + y^2 = z$, $x + y + z = 1$.

(h) Show that the differential equation $\left|\dfrac{dy}{dx}\right| + |y| = 0$ has a particular solution which is bounded.

(i) Obtain the singular solution of the differential equation $y - px - \dfrac{1}{p} = 0$, where $p = \dfrac{dy}{dx}$.

2. (a) If $P_n = D^n(x^n \log x)$ then prove that $P_n = nP_{n-1} + (n-1)!$. Hence prove that $P_n = n!(\log x + 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots + \frac{1}{n})$.   4

(b) If $x^{2/3} + y^{2/3} = c^{2/3}$ is the envelope of the lines $\frac{x}{a} + \frac{y}{b} = 1$ where a, b are variable parameters and c is a constant then prove that $a^2 + b^2 = c^2$.   4

3. (a) Prove that the length of the loop of the curve $x = t^2$, $y = t - \dfrac{t^3}{3}$ is $4\sqrt{3}$. 4

(b) Find the asymptotes of the curve $x^2(x+y)(x-y)^2 + 2x^3(x-y) - 4y^3 = 0$. 4

4. (a) Find the range of values of x for which the curve $y = x^4 - 16x^3 + 42x^2 + 12x + 1$ is concave or convex with respect to the x-axis and identify the points of inflexion if any. 4

(b) If $y = \sin(m \sin^{-1} x)$, show that $(1 - x^2)y_{n+2} - (2n+1)x\, y_{n+1} + (m^2 - n^2)\, y_n = 0$. 4

5. (a) Find the equation of the generating lines of the hyperboloid $3xy + yz + 2zx + 6 = 0$ which passes through the point $(-1, 0, 3)$. 4

(b) Reduce the equation $4x^2 + 4xy + y^2 - 4x - 2y + a = 0$ to the canonical form and determine the type of the conic for different values of a. 4

6. (a) Find the equation of the cone whose vertex is $(1, 0, -1)$ and which passes through the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$. 4

(b) Find the equation of the curve in which the plane $z = h$ cuts the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ and find the area enclosed by the curve. 4

7. (a) The section of the cone whose guiding curve is the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of the vertex is the surface $\dfrac{x^2}{a^2} + \dfrac{(y^2 + z^2)}{b^2} = 1$. 4

(b) Show that the equation of the circle, which passes through the focus of the parabola $\dfrac{2a}{r} = 1 + \cos\theta$ and touches it at a point $\theta = \alpha$, is given by $r\cos^3\dfrac{\alpha}{2} = a\cos(\theta - \dfrac{3}{2}\alpha)$. 4

8. (a) Show that the general solution of the equation $\dfrac{dy}{dx} + Py = Q$ can be written in the form $y = k(u - v) + v$ where k is a constant, u and v are its two particular solutions. 4

(b) Determine the curve in which the area enclosed between the tangent and the coordinate axes is equal to a^2. 4

9. (a) Solve $y(xy + 2x^2 y^2) dx + x(xy - x^2 y^2) dy = 0$.   4

(b) Reduce the equation $\sin y \dfrac{dy}{dx} = \cos x (2\cos y - \sin^2 x)$ to a linear equation and hence solve it.   4

10.(a) Using the transformation $u = x^2$ and $v = y^2$ to solve the equation $xyp^2 - (x^2 + y^2 - 1) p + xy = 0$, where $p = \dfrac{dy}{dx}$.   4

(b) Solve $(x^2 y^3 + 2xy) dy = dx$.   4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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