सत्यं शिवमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2023

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5 = 10$

(a) Verify the integrability of the following differential equation

$$3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0.$$

(b) Find an integrating factor of the differential equation $x\dfrac{dy}{dx}=x^2+3y,\ x>0$

(c) Construct a differential equation by elimination of the arbitrary constants a and b from the equation $z=ae^{-b^2t}\cos bx$.

(d) Show that x, x^2 and x^4 are linearly independent solutions of

$$x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=0.$$

(e) Show that the differential equation satisfied by the family of curves given by $c^2+2cy-x^2+1=0$, where c is the parameter of the family, is

$$(1-x^2)p^2+2xyp+x^2=0, \text{ where } p=\frac{dy}{dx}.$$

(f) Find the transformation of the differential equation

$$x^2\frac{d^2y}{dx^2}-5y=\log x$$

by using the substitution $x=e^z$.

(g) Determine the order, degree and linearity of the following P.D.E.

$$x\left(\frac{\partial z}{\partial x}\right)^2+xz\frac{\partial^2 z}{\partial x^2}-z\frac{\partial z}{\partial x}=0$$

(h) Form the partial differential equation by eliminating the arbitrary function from the following equation

$$z=F(x^2+y^2)$$

2. (a) Find the value of λ, for the differential equation $(xy^2+\lambda x^2y)dx+(x+y)x^2dy=0$ is exact. Solve the equation for this value of λ. 4

(b) Obtain the solution of the differential equation 4

$$xdy-ydx+a(x^2+y^2)dx=0$$

3. (a) Reduce the equation $(x^2y^3+2xy)dy = dx$ to a linear equation and solve it. 4

(b) If $y_1 = e^{-x}\cos x$, $y_2 = e^{-x}\sin x$ and $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$, then 2+1+1

(i) Calculate Wronskian determinant.

(ii) Verify that y_1 and y_2 satisfy the given differential equation.

(iii) Apply Wronskian test to check that y_1, y_2 are linearly independent.

4. (a) Solve $x\cos\left(\frac{y}{x}\right)(ydx+xdy) = y\sin\left(\frac{y}{x}\right)(xdy-ydx)$. 4

(b) Using the transformation $y^2 = Y$, $x = X$ to solve the equation $y = 2px - p^2y$ where $p = \frac{dy}{dx}$. 4

5. (a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y = \tan x$ 4

(b) Solve $\frac{dx}{dt}+5x+y=e^t$, $\frac{dy}{dt}+x+5y=e^{5t}$ 4

6. (a) Solve: $(D^3-2D^2-5D+6)y = (e^{2x}+3)^2+e^{3x}\cosh x$ where $D\equiv\frac{d}{dx}$. 4

(b) Reduce the equation $(px^2+y^2)(px+y) = (p+1)^2$ to Clairaut's form by substitutions $u = xy$, $v = x+y$, where $p = \frac{dy}{dx}$. Hence find its complete solution. 4

7. (a) Find the PDE of all sphere whose centre lie on *z*-axis and given by equations $x^2+y^2+(z-a)^2 = b^2$, *a* and *b* being constants. 4

(b) Solve: $(x^2-yz)p+(y^2-zx)q = z^2-xy$ 4

8. (a) Solve $px+qy = pq$ by Charpit's method. 4

(b) Solve: $\frac{\partial^3 z}{\partial x^3}-3\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$ 4

9. (a) Solve the P.D.E. $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2} = \sin x$. 4

(b) Determine the points (x, y) at which the partial differential equation 4

$(x^2-1)\frac{\partial^2 z}{\partial x^2}+2y\frac{\partial^2 z}{\partial x\partial y}-\frac{\partial^2 z}{\partial y^2} = 0$ is hyperbolic or parabolic or elliptic.

——×——

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2023

MTMACOR04T-MATHEMATICS (CC4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Explain, with the help of uniqueness and existence theorem, that the differential equation

$$\frac{dy}{dx}=\frac{y}{x}$$

has infinite number of solutions passing through the point (0, 0).

(b) Show that $e^x \sin x$ and $e^x \cos x$ are linearly independent solutions of the differential equation

$$\frac{d^2y}{dx^2}-2\cdot\frac{dy}{dx}+2y=0$$

(c) Solve $(D^2-4D)y=x^2$, $\left(D\equiv\frac{d}{dx}\right)$ by using the method of undetermined coefficients.

(d) Find the particular integral of the differential equation

$$(D^2-1)y=e^{-x},\ \left(D\equiv\frac{d}{dx}\right)$$

(e) Locate and classify the singular points of the equation

$$x^3(x-2)\frac{d^2y}{dx^2}-(x-2)\frac{dy}{dx}+3xy=0$$

(f) Find the magnitude of the volume of the parallelopiped having the vectors $\vec{a}=-3\hat{i}+7\hat{j}+5\hat{k}$, $\vec{b}=5\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{c}=7\hat{i}-5\hat{j}-3\hat{k}$ as the concurrent edges.

(g) If $\vec{F}=y\hat{i}-xz\hat{j}+x^2\hat{k}$ and C be the curve $x=t$, $y=2t^2$, $z=t^3$ from $t=0$ to $t=1$, then evaluate the integral $\int_C \vec{F}\times d\vec{r}$.

(h) A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\hat{i}+\sin\omega t\hat{j}$, where ω is a constant. Show that the acceleration $\vec{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin.

2. (a) If $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$ and $\vec{c}\times\vec{a}=\vec{b}$, then show that $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular. 4

(b) Show that in general $(\vec{a}\times\vec{b})\times\vec{c}\neq\vec{a}\times(\vec{b}\times\vec{c})$; but if the equality holds, then either $\vec{b}$ is parallel to $(\vec{a}\times\vec{c})$ or $\vec{a}$ and $\vec{c}$ are collinear. 4

3. (a) Integrate the function $\vec{F}=x^2\hat{i}-xy\hat{j}$ from (0, 0) to (1, 1) along the parabola $y^2=x$. 4

(b) Prove that the necessary and sufficient condition for the vector function $\vec{a}(t)$ to have constant magnitude is $\vec{a}\times\frac{d\vec{a}}{dt}=\vec{0}$. 4

4. (a) If $\vec{r}(t)=2\hat{i}-\hat{j}+2\hat{k}$ when $t=2$ and $\vec{r}(t)=4\hat{i}-2\hat{j}+3\hat{k}$ when $t=3$, then show that 4

$$\int_2^3\left(\vec{r}\cdot\frac{d\vec{r}}{dt}\right)dt=10$$

(b) Find the unit tangent, the curvature, the principal normal, the binormal and the torsion for the space curve 4

$$x=t-\frac{t^3}{3},\quad y=t^2,\quad z=t+\frac{t^3}{3}$$

5. (a) Solve $x^2\frac{d^2y}{dx^2}-3x\cdot\frac{dy}{dx}+y=\frac{\log_e x\sin\log_e x+1}{x}$. 4

(b) If y_1 and y_2 be two independent solutions of the linear homogeneous equation 4

$$\frac{d^2y}{dx^2}+P\cdot\frac{dy}{dx}+Q\cdot y=0$$

then show that the Wronskian $W(y_1, y_2)$ is given by

$$W(y_1, y_2)=A\cdot e^{-\int P\cdot dx},\ \text{where } A \text{ is a constant.}$$

6. (a) Solve the equation 4

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}=x+e^x\sin x$$

by the method of undetermined coefficients.

(b) Solve 4

$$\frac{d^2y}{dx^2}+2\cdot\frac{dy}{dx}+y=\frac{e^{-x}}{x^2}$$

by the method of variation of parameters.

7. (a) Solve 4

$$\frac{d^2x}{dt^2}+\frac{dy}{dt}+x+y=t \quad , \quad \frac{dy}{dt}+2x+y=0$$

given that $x=y=0$ at $t=0$.

(b) Solve: $\frac{d^2y}{dx^2}+4y=4\tan 2x$ 4

8. (a) Solve the equation $\frac{d^2y}{dx^2}+(x-1)^2\frac{dy}{dx}-4(x-1)y=0$ in series about the point $x=1$. 5

(b) Show that the point of infinity is a regular singular point of the equation 3

$$x^2\cdot\frac{d^2y}{dx^2}+(3x-1)\frac{dy}{dx}+3y=0$$

9. (a) Solve: $(D^3-1)y=\cos^2\frac{x}{2}$ 4

(b) Solve $x^2\cdot\frac{d^2y}{dx^2}+x\cdot\frac{dy}{dx}-y=0$, given that $x+\frac{1}{x}$ is one integral. 4

———×———

सत्यं शिवमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Examination, 2023

MTMACOR03T-MATHEMATICS (CC3)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) State Supremum property and Archimedean property of R, the set of all real numbers. 1+1

(b) Is the set $\{x \in R:\ \sin x \neq 0\}$ open in R? Justify your answer.

(c) Verify Bolzano-Weierstrass theorem for the set $\left\{\frac{n}{n+1} :\ n \in \mathbb{N}\right\}$.

(d) Prove that the sequence $\{x_n\}$ where $x_n = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \cdots\cdots + \frac{1}{(2n-1)(2n+1)}$ is bounded.

(e) If $A = [-1,\ 4)$ and $B = (2,\ 5]$, is $A \cup B$ compact? Give reasons.

(f) Show that the sequence $\{x_n\}$ is a null sequence where $x_n = \frac{n!}{n^n}$.

(g) Use comparison test to examine the convergence of the series:

$$\frac{1}{1\cdot 2^2} + \frac{1}{2\cdot 3^2} + \frac{1}{3\cdot 4^2} + \cdots\cdots$$

(h) Test the convergence of the series:

$$1 - \frac{2^2}{2!} + \frac{3^3}{3!} - \frac{4^4}{4!} + \cdots\cdots$$

2. (a) Let A and B be two non-empty bounded sets of real numbers. Let $C = \{x + y\ :\ x \in A,\ y \in B\}$. Show that $\sup C = \sup A + \sup B$. 3

(b) If S be a subset of R, then prove that interior of S is an open set. 2

(c) Prove that the set $\mathbb{Q}$ of rational numbers is enumerable. 3

3. (a) If $S = \{(-1)^m + \frac{1}{n}\ ;\ m \in N,\ n \in N\}$, then find the derived set of S. Is S a closed set? Justify your answer. 3

(b) If G is an open set in R then prove that $R - G$ is closed. 2

(c) Let $S = \bigcup_{n=1}^{\infty} I_n$, where $I_n = \left\{x \in R : \frac{1}{2^n} \leq x \leq 1\right\}$. Is the set S closed? Justify your answer. 3

4. (a) Prove that every compact subset of R is closed and bounded. 5

(b) Give an example of a set which is closed, but not compact. Give reasons. 1

(c) Prove that the intersection of two compact sets in R is compact. 2

5. (a) State and prove Sandwich theorem for convergence of a sequence and use it to prove that $\lim_{n\to\infty}(2^n+3^n)^{1/n}=3$. 1+2+2

(b) If $u_1>0$ and $u_{n+1}=\frac{1}{2}\left(u_n+\frac{9}{u_n}\right)$, $\forall\ n\geq 1$, then show that $\{u_n\}$ is monotonically decreasing and bounded below. Is it convergent? 3

6. (a) If a sequence $\{u_n\}$ converges to l, then prove that every subsequence of $\{u_n\}$ converges to l. 2

(b) If the n-th term of the sequence $\{u_n\}$ is given by $u_n=(-1)^n+\sin\frac{n\pi}{4}$, $n=1, 2, 3, \cdots\cdots$, then find two subsequences of $\{u_n\}$, one converging to the upper limit and the other converging to the lower limit. Is the sequence convergent? Give reasons. 3

(c) Show that $\lim_{n\to\infty}\frac{1\cdot 3\cdot 5\cdot\cdots\cdots(2n-1)}{2\cdot 4\cdot 6\cdot\cdots\cdots 2n}=0$. 3

7. (a) Prove that every convergent sequence is bounded. Is the converse true? Give reasons. 2+1

(b) Using definition of Cauchy sequence, show that the sequence $\{\frac{1}{n}\}$ is a Cauchy sequence. 2

(c) Prove or disprove: A monotone sequence of real numbers having a convergent subsequence is convergent. 3

8. (a) State and prove Leibnitz test for convergence of an alternating series. 1+3

(b) Use this to test the convergence of the series $\sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{n\log n}$. 2

(c) Define conditionally convergent series with example. 1+1

9. (a) Use Cauchy's integral test to show that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges for $p>1$ and diverges for $p\leq 1$. 3

(b) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}.(n!)^2.7^n}{(2n)!}$. 3

(c) If $\sum_{n=1}^{\infty}a_n$ is a convergent series of positive real numbers, will the series $\sum_{n=1}^{\infty}a_{2n}$ be convergent? Justify your answer. 2

———×———