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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2023

MTMACOR13T-MATHEMATICS (CC13)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Let (X,d) be a metric space and $A \subset X$. Then show that $\overline{A}$ is a closed set where $\overline{A}$ is the closure of A. 2

(b) Let $X=(0,1]$ be the metric space with usual metric and $\{x_n\}_n$ where $x_n=\frac{1}{n}$ be a sequence in X. Show that $\{x_n\}_n$ is a Cauchy sequence. Is X complete? Justify your answer. 2

(c) Give an example of a complete metric space and an incomplete metric space. 2

(d) Show that in a metric space any two disjoint sets are always separated. 2

(e) Evaluate $\int_0^{2+i} \overline{z}^2\, dz$ along the line $2y=x$. 2

(f) Find $\lim\limits_{z\to i}\dfrac{\overline{z}+z^2}{1-\overline{z}}$. 2

(g) Show that the function $f(z)=|z|^2$ is continuous everywhere on C. 2

(h) Prove that $f(z)=\operatorname{Re} z$ is nowhere differentiable. 2

2. (a) Let (X,d) be a metric space. Then show that $\sigma(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$ is a bounded metric on X, $x, y\in X$. 5

(b) Prove that every Cauchy sequence in a metric space is bounded. 3

3. (a) State and prove Cantor's intersection theorem. 4

(b) Let $\{x_n\}$, $\{y_n\}$ be two convergent sequences in a metric space (X,d) and converge to $x, y\in X$ respectively then prove that the sequence $\{d(x_n, y_n)\}$ of real numbers converges to $d(x,y)$ in usual real metric space. 4

4. (a) Let (X, d_1) and (Y, d_2) be two metric spaces and $f : X \to Y$ be a function. Then show that f is continuous at a point $x_0 \in X$ if and only if $f(x_n) \to f(x_0)$ for every sequence $\{x_n\}_n \subset X$ with $x_n \to x_0$. 4

(b) Let X, Y be two metric spaces. If $f : X \to Y$ is continuous then show that for every compact subset $E \subset X$, the image $f(E)$ is a compact subset of Y. 4

5. (a) Let (X, d) be a metric space, $G \subset X$ and $G = A \cup B$ where A and B are separated sets. Show that if G is open then A and B are open. 4

(b) Prove that $A \subset \mathbb{R}$ is connected with respect to usual metric if and only if it is an interval. 4

6. (a) Let $f : D \longrightarrow \mathbb{C}$ $(D \subset \mathbb{C})$ be a complex valued function and $z_0 \in D$, where $f(z) = u(x, y) + i\,v(x, y)$ $(x, y \in \mathbb{R})$. Then show that the function $f(z)$ is continuous at $z_0 = x_0 + iy_0$ iff $u(x, y)$ and $v(x, y)$ are continuous at (x_0, y_0). 4

(b) If the complex sequence $\{z_n\}_n$ where $z_n = a_n + ib_n$, $n \in \mathbb{N}$ converges to $z = a + ib$, then prove that $\{|z_n|\}_n$ converges to $|z|$. Is the converse of the above result true? Justify your answer. 2+2

7. (a) Let $f(z) = u(x, y) + i\,v(x, y)$ be a function defined in a region D such that u, v and their first order partial derivatives are continuous in D and first order partial derivative of u and v satisfy Cauchy-Riemann equations at a point $(x, y) \in D$, then prove that f is differentiable at $z = x + iy$. 4

(b) Show that the function $f(z) = u + iv$ where 4

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} \qquad z \neq 0$$
$$= 0 \qquad z = 0$$

is not differentiable at the origin even though it satisfy Cauchy-Riemann equations at the origin.

8. (a) State and prove Cauchy's integral formula for the first order derivative of an analytic function. 5

(b) Show that $\oint_{\gamma} \frac{z+4}{z^2 + 2z + 5}\, dz = 0$. 3

9. (a) If f is an integral function and $|f(z)| < M$, for all z, M being a positive constant then prove that f is constant. 4

(b) Find Taylor series expansion of $f(z) = \frac{z-1}{(z+1)}$ 4

(i) about the point $z = 0$

(ii) about the point $z = 1$.

Determine the region of convergence in each case.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2023

MTMADSE04T-MATHEMATICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Show that the equation $x^n+nx^{n-1}+n(n-1)x^{n-2}+\ldots+n!=0$ can not have equal roots.

(b) If $\alpha,\beta,\gamma,\delta$ are the roots of $x^4+px^3+qx^2+rx+s=0$ then find the value of $\sum\alpha^2\beta\gamma$.

(c) If α is a special root of $x^n-1=0$, then prove that $\frac{1}{\alpha}$ is also a special root of it.

(d) If α be a imaginary root of the equation $x^n-1=0$, and n be a prime number, then show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)\ldots.(1-\alpha^{n-1})=n$.

(e) Find the value of k for which $(x+1)^4+x^4+\frac{k}{2}=0$ is a reciprocal equation.

(f) If α be a special root of the equation $x^8-1=0$, prove that

$$1+3\alpha+5\alpha^2+\ldots\ldots\ldots\ldots+15\alpha^7=\frac{16}{\alpha-1}$$

(g) Show that for real values of μ, the equation $(x+1)(x-3)(x-5)(x-7)+\mu(x-2)(x-4)(x-6)(x-8)=0$ has all its roots real and simple.

(h) Find the equation of fourth degree with real coefficient one root of which is $\sqrt{2+i\sqrt{3}}$.

2. (a) Find the special roots of the equation $x^{24}-1=0$ and hence deduce that $\cos\frac{\pi}{12}=\frac{\sqrt{3}+1}{2\sqrt{2}}$ and $\cos\frac{5\pi}{12}=\frac{\sqrt{3}-1}{2\sqrt{2}}$. 4

(b) A polynomial $f(x)$ leaves the remainders 10 and $2x-3$ when it is divided by $(x-2)$ and $(x+1)^2$ respectively. Find the remainder when it is divided by $(x-2)(x+1)^2$. 4

3. (a) Show that the equation $x^4-14x^2+24x+k=0$ has two distinct real roots if $-8<k<117$. 4

   (b) Applying Strum's theorem show that the equation $x^4-12x^2+4=0$ has all roots are real and distinct. 4

4. (a) Show that the roots of the equation $\frac{1}{x+a_1}+\frac{1}{x+a_2}+\cdots+\frac{1}{x+a_n}=\frac{1}{x+b}$ are all real, where $a_1, a_2, \ldots a_n$ and b are all positive real numbers and $b>a_i$ for all i. 4

   (b) Solve the cubic $x^3-27x-54=0$ by Cardan's method. 4

5. (a) Applying Newton's theorem find the sum of 5^{th} powers of the roots of the equation $x^3+qx+r=0$. 4

   (b) If the equation $x^5-10a^3x^2+b^4x+c^5=0$ has three equal roots, then show that $ab^4-9a^5+c^5=0$. 4

6. (a) If the two polynomials $f(x)$ and $g(x)$, both of degree n take equal values for more than n distinct values of x, then $f(x)$ and $g(x)$ are identical polynomials. 3

   (b) Solve the equation $x^4+5x^3+x^2-13x+6=0$ by Ferrari's method. 5

7. (a) Solve $x^5-5x^4+9x^3-9x^2+5x-1=0$. 4

   (b) If n be a prime number prove that the special roots of the equation $x^{2n}-1=0$ are the non real roots of the equation $x^n+1=0$. 4

8. (a) Show that the condition that the sum of two roots of the equation $x^4+mx^2+nx+p=0$ is equal to the product of the other two roots is $(2p-n)^2=(p-n)(p+m-n)^2$. 4

   (b) Find the values of k for which the equation $x^3-9x^2+24x+k=0$ may have multiple roots and solve the equation in each case. 4

9. (a) Find an upper limit of the real roots of the equation $x^4+4x^3-11x^2-9x-50=0$. 4

   (b) If α, β, γ be the roots of $x^3+qx+r=0$, prove that $6S_5=5S_2S_3$, where $S_r=\Sigma\alpha^r$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 6th Semester Examination, 2023

MTMGDSE04T-MATHEMATICS (DSE2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Find the extreme points of the convex set $X=\{(x, y): x^2+y^2 \leq 4\}$.

(b) Examine whether (4, 0, 3, 0) is a basic feasible solution of the equations

$$x_1+x_2-x_3+2x_4=1$$
$$2x_1+x_2-2x_3-4x_4=2.$$

(c) Draw the feasible region of the following LPP.

Maximize $Z=-2x_1+x_2$

Subject to $x_1+x_2 \leq 6$

$3x_1+2x_2 \leq 12$

$x_1, x_2 \geq 0$.

(d) Why do we use minimum ratio criterion in Simplex method?

(e) Find the dual of the following primal problem

Maximize $Z=3x_1-2x_2$

Subject to $2x_1+x_2 \leq 1$

$-x_1+3x_2 \geq 4$

$x_1, x_2 \geq 0$

(f) Find a supporting hyperplane of the set $X=\{(x, y): y^2 \leq 4x\}$.

(g) Introducing slack and surplus variables, write the following problem in a standard form:

Maximize $Z=x_1-x_2+2x_3$

Subject to $2x_1-x_2+3x_3 \leq 6$

$x_1+4x_2-5x_3 \leq -5$

$x_1, x_2, x_3 \geq 0$.

(h) Define convex hull and convex polyhedron.

2. (a) An agricultural farm has 180 tons of Nitrogen fertilizers, 250 tons of phosphate and 220 tons of potash. It is able to sell 3:3:4 mixtures of these substances at a profit of Rs. 15 per ton and 1:2:1 mixtures at profit of Rs. 12 per ton respectively. Formulate a linear programming problem to show how many tons of these two mixture should be prepared to obtain the maximum profit. 4

(b) Solve the following L.P.P. using graphical method 4

Maximize $Z = 6x_1 + 4x_2$

Subject to $7x_1 + 5x_2 \leq 35$,

$5x_1 + 7x_2 \leq 35$,

$4x_1 + 3x_2 \geq 12$,

$3x_1 + x_2 \geq 3$

and $x_1, x_2 \geq 0$.

3. (a) Find all the basic feasible solutions of the system: 4

$2x_1 + 6x_2 + 2x_3 + x_4 = 3$

$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$.

State that whether the B.F.S are non-degenerated or degenerated.

(b) If x_1, x_2 be real, show that the set given by $X = \{(x_1, x_2) : 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set. 4

4. (a) Solve the following LPP: 4

Maximize $Z = 60x_1 + 50x_2$

Subject to $x_1 + 2x_2 \leq 40$

$3x_1 + 2x_2 \leq 60$

$x_1, x_2 \geq 0$.

(b) Prove that if the L.P.P. has at least two optimal feasible solution then there are infinite number of optimal solutions. 4

5. (a) Use two-phase method to solve the following L.P.P. 5

Maximize $Z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2$

$x_1 \leq 4$

$x_1, x_2 \geq 0$

(b) Prove that if any variable of the primal problem be unrestricted in sign, then the corresponding constraint of the dual will be equality. 3

6. (a) Solve the following L.P.P. by Big M-method: 6

Minimize $Z = 2x_1 + 9x_2 + x_3$

Subject to $x_1 + 4x_2 + 2x_3 \geq 5$,

$3x_1 + x_2 + 2x_3 \geq 4$,

and $x_1, x_2, x_3 \geq 0$

(b) Why do we introduce artificial variable in solving L.P.P.? 2

7. (a) If the feasible region of a linear programming problem is strictly bounded and contains a finite number of extreme points then prove that the objective function of the linear programming problem assumes its optimal value at an extreme point of the convex set of feasible solutions. 4

(b) Show that the feasible solution $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 2$ to the system 4

$$x_1 + x_2 + x_3 = 2$$
$$x_1 + x_2 - 3x_3 = 2$$
$$2x_1 + 4x_2 + 3x_3 - x_4 = 4$$
$$x_1, x_2, x_3, x_4 \geq 0$$

is not basic.

8. (a) If x be any feasible solution of the primal problem Max $Z = cx$, $Ax \leq b$, $x \geq 0$; and v be any feasible solution of its dual problem then prove that $cx \leq b^T v$. 4

(b) Find the dual of the following primal 4

Maximize $Z = x_1 + 2x_2 + 3x_3$

Subject to $x_1 + 2x_2 + x_3 = 4$

$$x_1 - x_2 \leq 2$$
$$3x_2 + x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0$$

9. (a) Show that (2, 1, 3) is a feasible solution of the system of equations: 5

$$4x_1 + 2x_2 - 3x_3 = 1$$
$$-6x_1 - 4x_2 + 5x_3 = -1$$
$$x_1, x_2, x_3 \geq 0.$$

Reduce the feasible solution to a basic feasible solution.

(b) Prove that the set of all feasible solutions of a linear programming problem is a convex set. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2023

MTMADSE06T-MATHEMATICS (DSE3/4)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Define Virial of coplanar forces acting on a body.

(b) If two forces act on a particle and the particle undergoes a small displacement, show that the total work done by those forces is equal to the work done by their resultant.

(c) When frictional force arises between two static rough bodies, in contact with each other, then what is the relation between the angle of friction and the co-efficient of friction?

(d) If V is the potential energy of a body in equilibrium under the action of some forces, state the conditions indicating the equilibrium is stable or unstable.

(e) If the moment of inertia of a circular disc of mass M and radius a about any of its diameter is $\frac{Ma^2}{4}$, find the moment of inertia about a line passing through the centre and perpendicular to the plane of the disc.

(f) Define kinetically equivalent systems. State the necessary and sufficient condition that two systems be kinetically equivalent.

(g) If a rigid body as compound pendulum swings under gravity about a fixed horizontal axis, then write the expression of the length of equivalent pendulum. Define all the terms correctly.

(h) Find the velocity of an artificial satellite of the earth, given $g=9.8\text{ m/s}^2$ and radius of the earth $=6.4\times10^6$ m.

2. (a) Two uniform rods, AB and CD each of weight 'W' and length 'a' are smoothly jointed at O where $OB=OD=b$. The rods rest in a vertical plane with the ends A and C on a smooth table and the ends B and D are connected by a light string. Show that the reaction at the joint is $\frac{aW\tan\alpha}{2b}$, where α is the inclination of either rod to the vertical. 4

(b) Find the centre of gravity of the arc of the cardioid $r=a(1+\cos\theta)$ lying above the initial line. 4

3. (a) Two equal uniform rods of length l jointed at one end so that the angle between them is θ and they rest in a vertical plane on a smooth sphere of radius R. Show that the rods are in unstable or stable equilibrium according as 4

$$l \lessgtr 4R\operatorname{cosec}\theta$$

(b) A semi-circular disc rests in a vertical plane with its curved edge on a rough horizontal plane and rough vertical plane, where μ and μ' are the coefficients of frictions at the horizontal plane and vertical plane respectively. Show that the greatest angle that the bounding diameter makes with the vertical plane is 4

$$\cos^{-1}\left(\frac{3\mu\pi}{4}\cdot\frac{1+\mu'}{1+\mu\mu'}\right).$$

4. (a) Find the equation of Poinsot's central axis for any system of forces in three dimensions. 4

(b) A force F acts along the axis of z, and a force mF along a straight line, intersecting the axis of x at a distance c from the origin and parallel to the plane of yz. Show that as this straight line turns around the axis of x, the central axis of the forces generate the surface $\{m^2z^2+(m^2-1)y^2\}(c-x)^2 = x^2z^2$. 4

5. (a) Deduce the differential equation of a central orbit under a central force in pedal form. 4

(b) If a particle describes a nearly circular path of radius $1/c$ under the influence of a central force $\mu\varphi(u)$ (where, $u=\frac{1}{r}$, r being the distance of the particle at any instant from the centre of force), find the condition that this may be a stable motion. 4

6. (a) If a planet were suddenly stopped in its orbit, supposed circular, then show that it would fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. 4

(b) The motion of a point relative to a fixed frame is defined by $x = a\cos\omega t$, $y = b\sin\omega t$. Show that the motion of the point represented in a moving frame with the same origin will describe a circle, if the frame revolves in a positive sense with angular velocity ω. 4

7. Find whether a straight line is at any point of its length, a principal axis of a given material system. If so, find the direction of the other two principal axes. Hence show that through each point of a plane lamina there exists a pair of principal axes of the lamina. 8

8. (a) Show that the kinetic energy of a rigid body of mass M moving in two dimensions is equal to the sum of kinetic energy of a particle of mass M placed at the centre of inertia and moving with it and the kinetic energy of the body relative to the centre of inertia. 4

(b) A uniform rod of length $2a$ is placed with one end in contact with a smooth horizontal table and is then allowed to fall; if α be its initial inclination to the vertical, show that its angular velocity is 4

$$\sqrt{\frac{6g}{a}\cdot\frac{\cos\alpha-\cos\theta}{1+3\sin^2\theta}},$$

when it is inclined at angle θ.

9. (a) A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding. Show that the centre of the sphere moves with a constant acceleration $\frac{5}{7}g\sin\alpha$ down the plane and for pure rolling $\mu>\frac{2}{7}\tan\alpha$, 5

where α is the inclination of the plane to the horizontal, μ is the coefficient of friction and g is the acceleration due to gravity.

(b) A heavy circular disc is revolving in a horizontal plane about its centre, which is fixed. An insect of mass $\frac{1}{n}$th that of the disc walks from the centre along a radius and then flies away. Show that the final angular velocity is $\frac{n}{n+2}$ times the original angular velocity of the disc. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2023

MTMACOR14T-MATHEMATICS (CC14)

RING THEORY AND LINEAR ALGEBRA II

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 =10

(a) In the ring $\mathbb{Z}_8[x]$, show that $[1]+[2]x$ is a unit.

(b) Let R be an integral domain and p be a prime element in R. Then show that p is irreducible.

(c) Prove that [2] is not an irreducible element in $\mathbb{Z}_6$.

(d) Find all associates of $1+i$ in $Z[i]$.

(e) Let T be a linear operator on $V=\mathbb{R}^2$ defined by $T(a,b)=(-2a+3b,-10a+9b)$. Find the eigen values of T.

(f) Let S be a subset of a vector space V over a field F. Show that S^0 is a subspace of V^*, where S^0 denotes the annihilator of S and V^* denotes the dual space of V.

(g) Let $\langle\ ,\rangle$ be the standard inner product on $\mathbb{R}^2$. Let $\alpha=(1,2)$ and $\beta=(-1,1)$. If γ is a vector such that $\langle\alpha,\gamma\rangle=-1$ and $\langle\beta,\gamma\rangle=3$, find γ.

(h) Let V be a finite dimensional vector space. What is the minimal polynomial of the identity operator on V?

2. (a) If D is an integral domain, show that $D[x]$ is an integral domain. Also show that if D is a field, then $D[x]$ can never be a field. 3+1

(b) Let F be a field and $\alpha: F[x]\to F[x]$ be an automorphism such that $\alpha(a)=a$ for all $a\in F$. Show that $\alpha(x)=ax+b$ for some $a,b\in F$. 4

3. (a) Prove that every irreducible element in a UFD is a prime element. 4

(b) Test for the irreducibility of the following polynomials: 2+2

(i) $x^3-[9]$ over $\mathbb{Z}_{11}$

(ii) $x^4+x^3+x^2+x+1$ over $\mathbb{Z}$.

4. (a) Prove that every Euclidean domain is a PID. 4

(b) Find a gcd of the elements $3+i, 5+i$ in the Euclidean domain $Z[i]$ with a Euclidean valuation v defined by $v(m+ni)=m^2+n^2$ for $m+ni \in Z[i]$. If d is the gcd, express d as $d=(3+i)u+(5+i)v$ for some u, v in $Z[i]$. 4

5. (a) Find the dual basis for the ordered basis 4

$$\mathcal{B}=\{(1,-2,3),(1,-1,1),(2,-4,7)\} \text{ of } V_3(\mathbb{R}).$$

(b) Let V be a finite dimensional vector space over a field $\mathbb{F}$. Define $\psi: V \to V^{**}$ by $\psi(x)=\hat{x}, \forall x \in V$, where $\hat{x}: V^* \to \mathbb{F}$ is defined by $\hat{x}(f)=f(x), \forall f \in V^*$. Show that ψ is an isomorphism from V to V^{**}. 4

6. (a) Apply Gram-Schmidt process to the vectors $\beta_1=(1,0,1)$, $\beta_2=(1,0,-1)$, $\beta_3=(0,3,4)$, to obtain an orthonormal basis of $\mathbb{R}^3$ with the standard inner product. 4

(b) Find a matrix P such that $P^{-1}AP$ is in Jordan Canonical form, where 4

$$A=\begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 1 & 2 \end{pmatrix}.$$

7. (a) Let W be a subset of a vector space V over a field K. Define the *annihilator* of W. If U and W are subspaces of a vector space V over a field K, then show that $(U+W)^0=U^0 \cap W^0$, where U^0, W^0 are annihilators of U, W respectively. 1+4

(b) If W is a subspace of R^4, generated by (1, 2, 3, 4) and (1, 1, 1, 1), then find a basis of the annihilator of W. 3

8. (a) Let T be a linear operator on a complex inner product space V. Prove that T is normal if and only if $\| T^*(u) \| = \| T(u) \|, \ \forall u \in V$. 4

(b) Prove that the matrix $A=\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable over the field C. 4

9. Let $V=P_3(\mathbb{R})$ and T be a linear operator on V defined by $T(p(x))=xp'(x)+p''(x)-p(2)$. $\mathcal{B}=\{1, x, x^2, x^3\}$ is the standard ordered basis for V. Find the matrix representation of T relative to the basis $\mathcal{B}$ for V. Find the characteristic polynomial of $[T]_{\mathcal{B}}$. Show that T is diagonalizable. Also find the minimal polynomial of T. 1+2+4+1

(Here $p'(x)$ and $p''(x)$ denote the first and second order derivative of $p(x)$)

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