नास्त्रं त्रिश्यामम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2023

STSHGEC02T/STSGCOR02T-STATISTICS (GE2/DSC2)

INTRODUCTION TO PROBABILITY

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer any *four* questions from the following $5\times 4 = 20$

1. (a) If $P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{4}$, find $P(A\cup B^c)$.

   (b) Show that for any two events A and B, $P(A\mid B)\leq \dfrac{P(A)}{P(B)}$.

2. State and prove the Chebyshev's inequality. 2+3

3. (a) For a random variable X, show that $[E(X^2)]^{1/2}\geq E(X)$.

   (b) For what value of a, the following function is the pmf of a random variable X.

$$f(x)=a\left(\tfrac{1}{2}\right)^x \quad , \qquad x=0,1,2,\ldots\ldots$$
$$= 0 \quad , \qquad \text{otherwise}$$

4. For a binomial distribution, the mean and s.d. are respectively 4 and $\sqrt{3}$. Calculate the probability of getting a non-zero value from this distribution.

5. If the events E_1, E_2, E_3 are independent with $P(E_i^c)=\dfrac{i}{i+1}$, $i=1,2,3$, then find the probability that at least one of the three events occurs.

6. What is convergence in probability? State WLLN. $2\frac{1}{2}+2\frac{1}{2}$

7. (a) What is 'Standard Normal distribution' with some of its important properties? 3+2

   (b) X follows a normal distribution with mean 12 and standard distribution 4. Find $P(X\geq 20)$. Given $\int_{-\infty}^{2}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}\,dt=0.9772$.

8. What is a cumulative distribution function of a random variable? Derive cdf of exponential distribution starting from its probability density function.

Answer any *two* from the following questions $10\times2=20$

9. (a) A random variable X follows Poisson distribution with parameter m, show that, $P(X \text{ is even}) = \frac{1}{2}(1+e^{-2m})$. 4+4+2

(b) Determine $f(x)$, the probability mass function from $f(x)=\frac{\lambda}{x}f(x-1)$, $x=1, 2, \ldots\ldots$, where $f(x)$ is non-negative integral values of random variable X. Find also the probability that X is greater than zero.

10. Find the m.g.f. of Normal distribution with parameters μ and σ^2 and find mean, variance, measure of skewness and measure of kurtosis. 10

11.(a) For two variables X and Y, $E(X)=8$, $E(Y)=6$, $\text{var}(X)=16$, $\text{var}(Y)=36$ and $r_{xy}=0.5$. Find 6+4

(i) $E(XY)$

(ii) $\text{cov}(X, X+Y)$

(iii) $\text{var}(2X-2Y)$

(iv) Correlation coefficient between $(2X+3Y)$ and $(2X-3Y)$.

(b) Prove that, for any two independent random variables X and Y, $E(XY)=E(X)E(Y)$.

12. State and prove Bayes' theorem in probability. 10

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